



The Numerator – From Westergaard to ISLAB: Why Finite Element Modeling Matters for Concrete Pavements

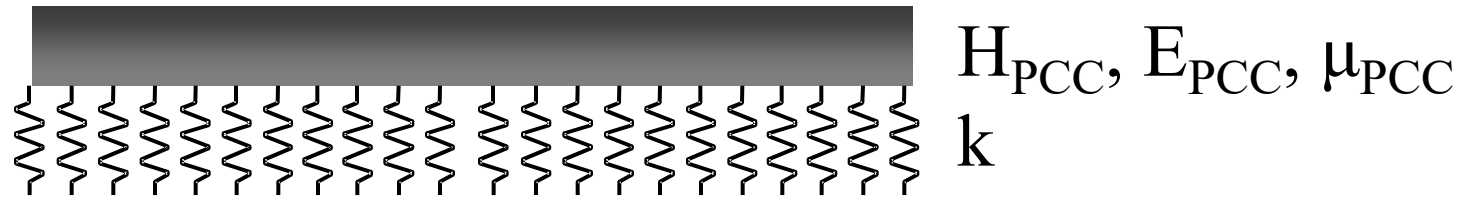
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Anthony Gill Chair Professor
Department of Civil and Environmental Engineering
University of Pittsburgh

Outline

- Westergaard model & solutions
- Limitations
- Evolution of FE programs
- ISLAB2000 capabilities
- Demonstration of results
- Why ISLAB is preferred

Westergaard Model

- PCC layer: a plate
- Subgrade : Winkler foundation



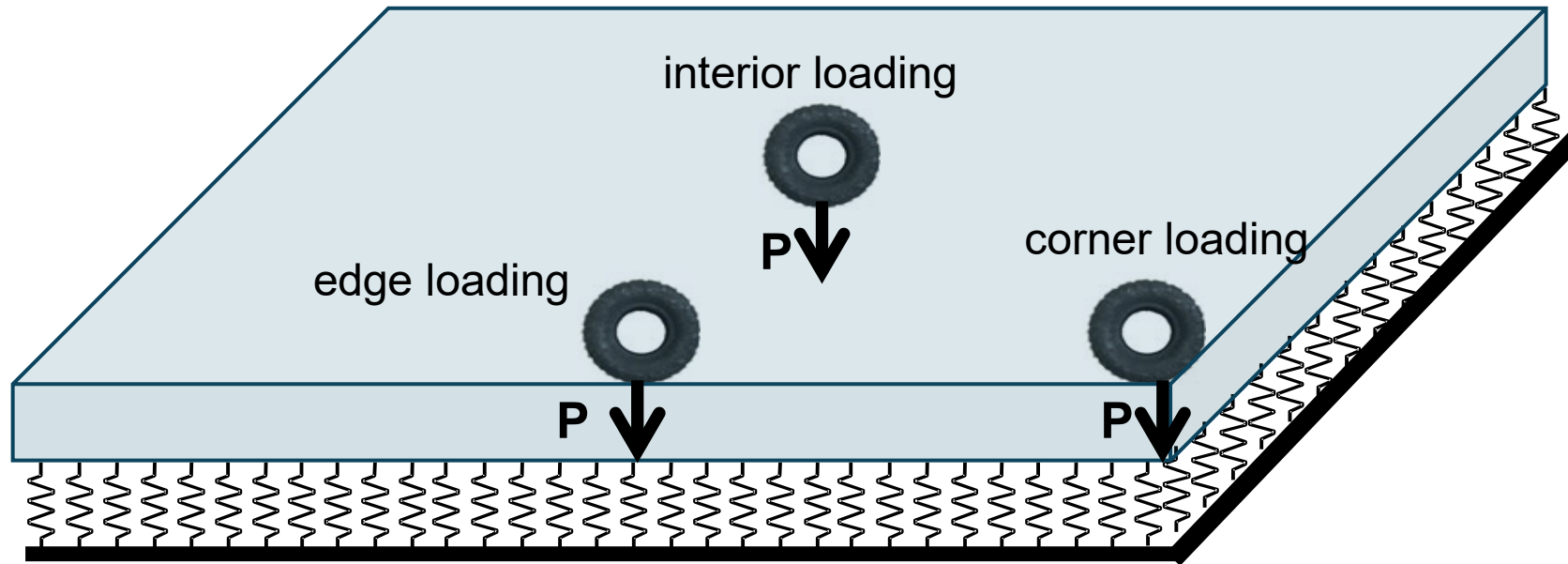
$$D \nabla^4 w(x, y) + k w(x, y) = p(x, y)$$

$$D = \frac{E_{PCC} h_{PCC}^3}{12(1 - \mu_{PCC}^2)}$$

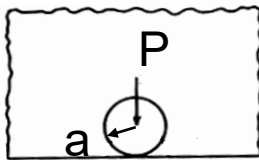
- Scale: radius of relative stiffness

$$\ell = \sqrt[4]{\frac{D}{k}}$$

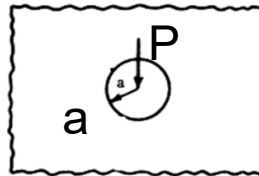
Westergaard's Solutions



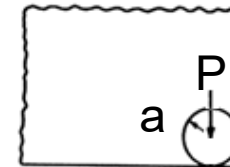
edge loading



interior loading



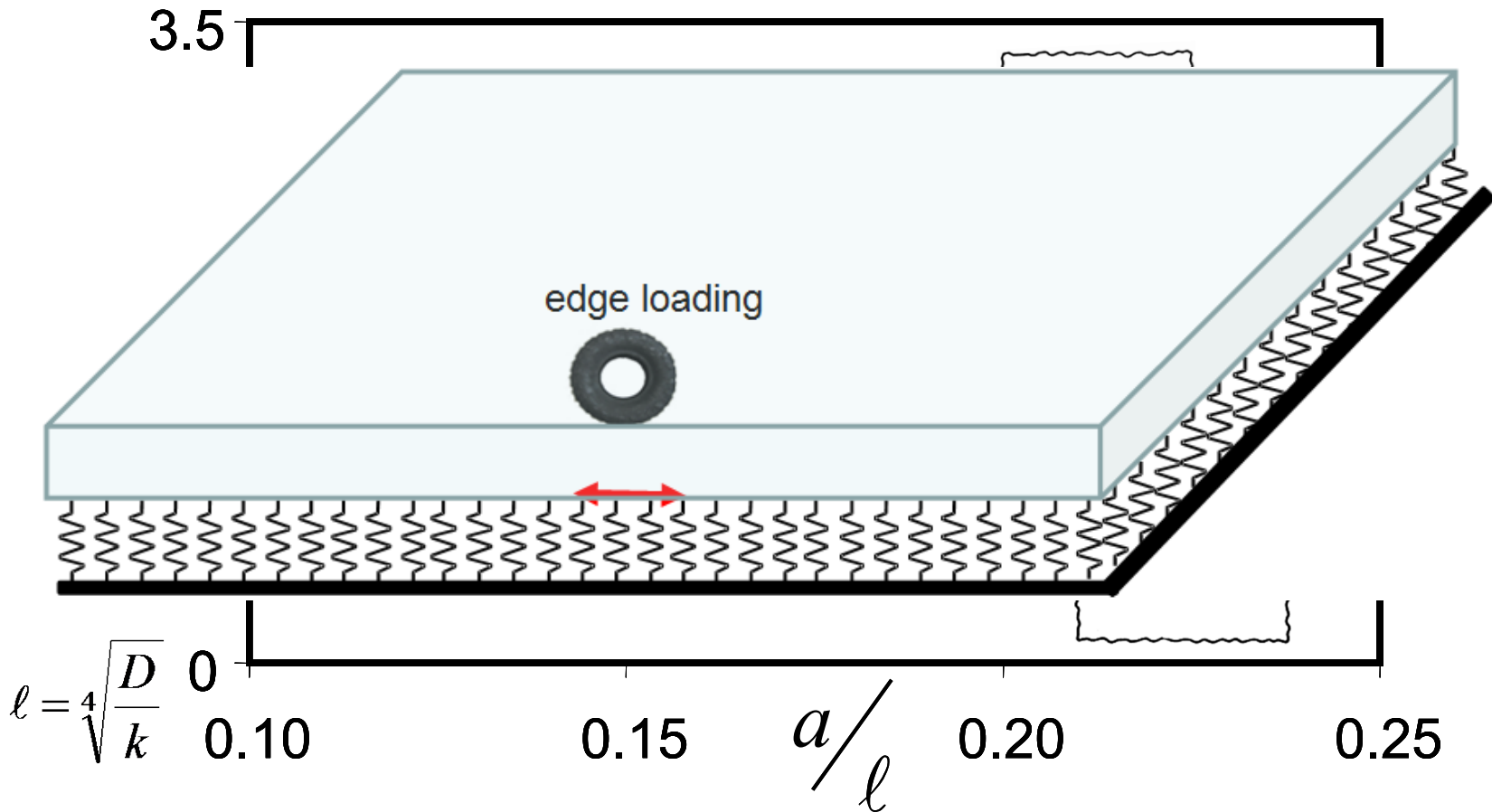
corner loading



a = wheel footprint radius, P = wheel load

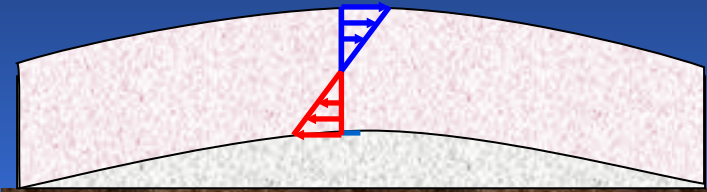
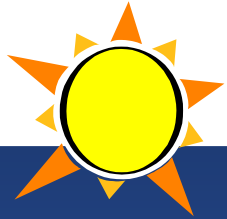
Westergaard's Solutions

Maximum Plate Tensile Stresses



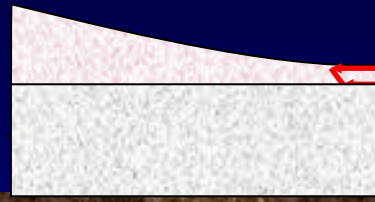
a = wheel footprint radius; P = wheel load; h = plate thickness

Curling and Warping



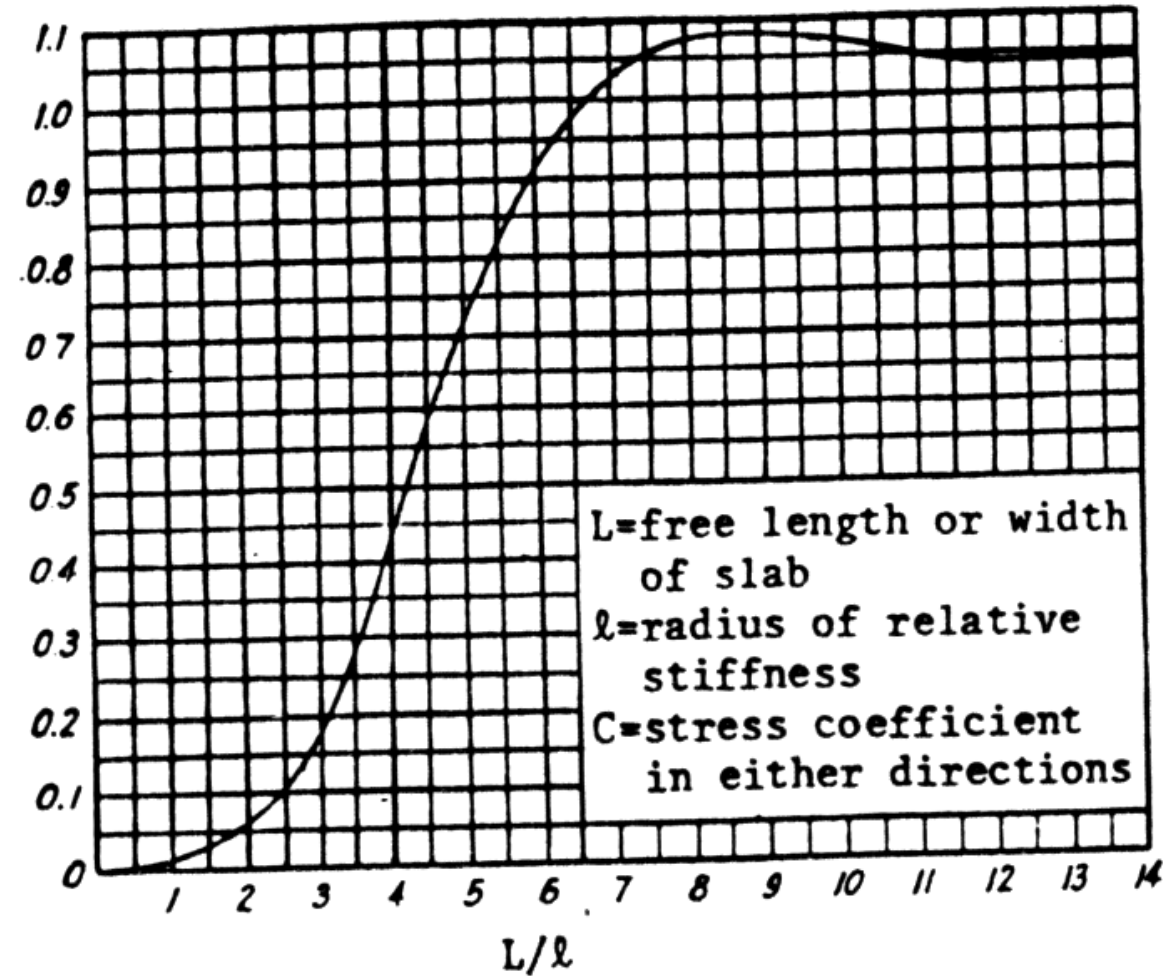
Foundation: Base and Subgrade

Positive temp. gradient



Foundation: Base and Subgrade

Negative temp. gradient



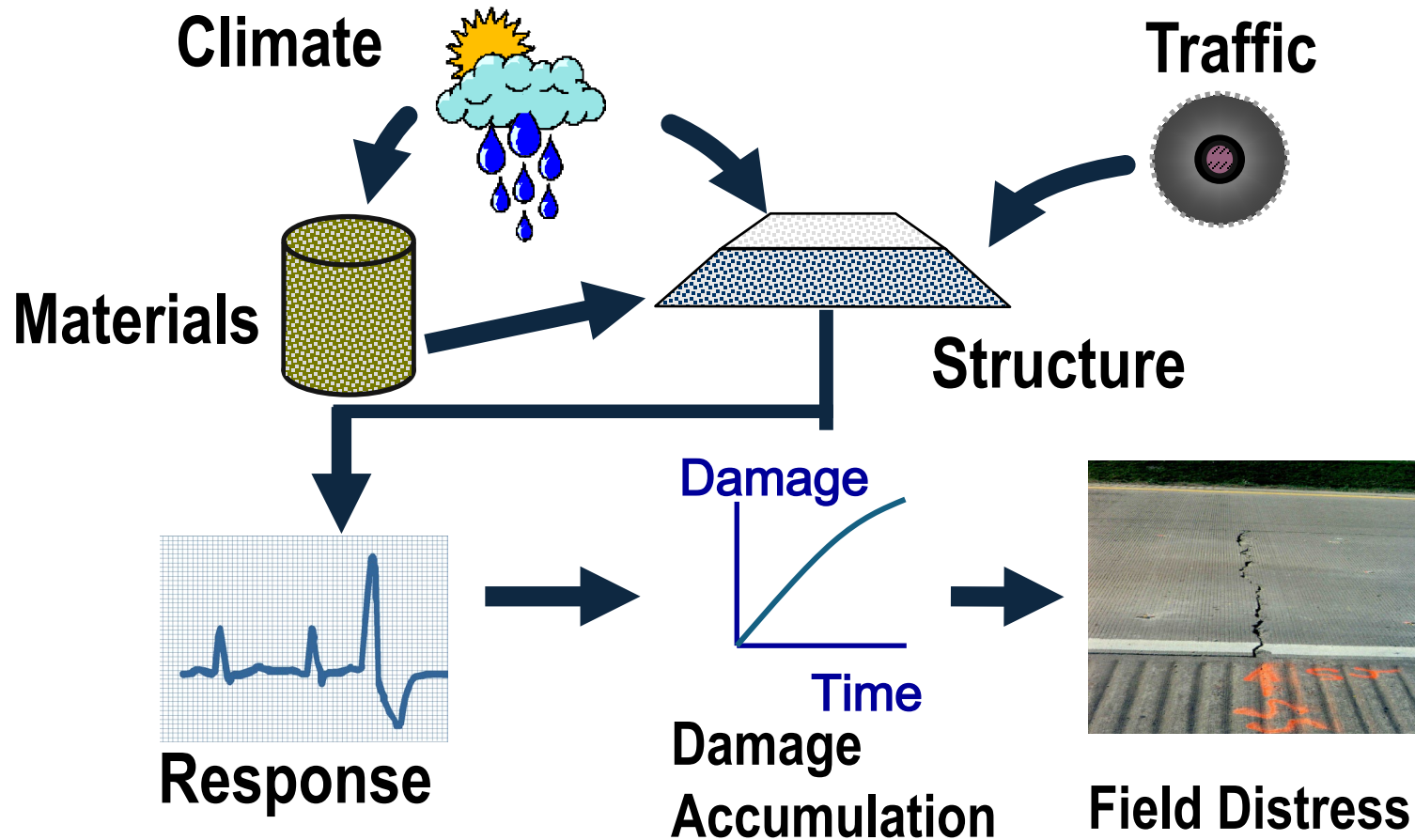
Limitations of Westergaard's Solutions

- Single slab
 - no joints
- Infinite/semi -infinite in horizontal direction
 - no slab size effect
- Single layer slab
- Full contact of the plate with the foundation
 - no separation
- Single wheel
 - no axle loads

Finite Element Programs

- KenSlab (Huang 1973), JSLAB (Tayabji 1977), EverFE (1999)
- ILLISLAB
 - Tabatabai and Barenberg (1978) – multiple slabs, base layer, doweled joints
 - Ioannides (1985) – Pasternak and elastic half-space foundations
 - Korovesis (1989) – slab curling, separation from subgrade
 - Khazanovich (1994) – nonlinear temperature distribution, separation between slab and base
 - Roesler and Khazanovich (1997) – fracture mechanics-based modeling of partial depth cracks
- ISLAB2000 (Khazanovich et al. 2000)

NCHRP 1-37A Project: Development of a MEPDG (1997 -2004)



Dr. Mike Darter
Professor (emeritus)
University of Illinois
Founder of ERES
Consulting, Inc.
NCHRP 1-37A Rigid
Pavements Group Leader

ISLAB2000 became an important mechanistic engine underlying the rigid-pavement analysis framework developed for MEPDG.

(NCHRP 2004, AASHTO 2008)

ISLAB2000

Suite of three programs:

- ISLAB2000Pre – a user-friendly preprocessor for development of ISLAB2000 finite element models
- ISLAB2000 – a computational engine performing the finite element analysis
- ISLAB2000Post – a tool for visualization of the ISLAB2000 analysis results

ISLAB2000Pre

- Define pavement geometry
- Provide characteristics of the pavement layers
- Provide subgrade properties
- Provide characteristics of joints
- Define the axle load
- Provide temperature distribution through the slab thickness

ISLAB2000Pre

- Geometry: Multiple slabs in both direction. Simple rules to generate mesh.
- Pavement structure: multiple layers, bonded and unbonded, possible separation between layers
- Subgrade: Winkler foundation, Pasternak, Kerr
- Load: multiple rectangular footprints
- Joints: Aggregate interlock; dowels, partial depth cracks
- Curling and Warping: Linear, Quadratic, and Piecewise Linear

Pavement Geometry

- Number of slabs in each direction and their lengths and widths
- Coordinates of nodes of the finite element mesh

The screenshot shows the 'Geometry' window of 'The Slab Designer' software. It is divided into two main sections: 'X-direction' and 'Y-direction'. Each section contains a table with columns for 'Slab', 'Length', and 'Number of Nodes'. In the 'X-direction' table, Slab 1 has a length of 180.0 and 46 nodes, Slab 2 has a length of 180.0 and 46 nodes, and Slab 3 has a length of 180.0 and 46 nodes. The 'Y-direction' table has identical data. To the right of these tables is a 'Symmetric' checkbox and a diagram showing a coordinate system with 'X' and 'y' axes. Below the 'Symmetric' checkbox is a 'Mesh' section with three radio buttons: 'Coarse', 'Medium', and 'Fine'. The 'Fine' radio button is selected. Below the 'Mesh' section is a 'Customize' button. At the bottom of the window, there is a 'Nominal Element Size' field set to '4', a 'Generate' button, a 'Mesh' button, and an 'OK' button. A traffic light icon is located at the bottom left of the window.

X-direction		
Slab	Length	Number of Nodes
1	180.0	46
2	180.0	46
3	180.0	46

Y-direction		
Slab	Length	Number of Nodes
1	180.0	46
2	180.0	46
3	180.0	46

Symmetric ☐

Mesh
☐ Coarse
☐ Medium
☒ Fine

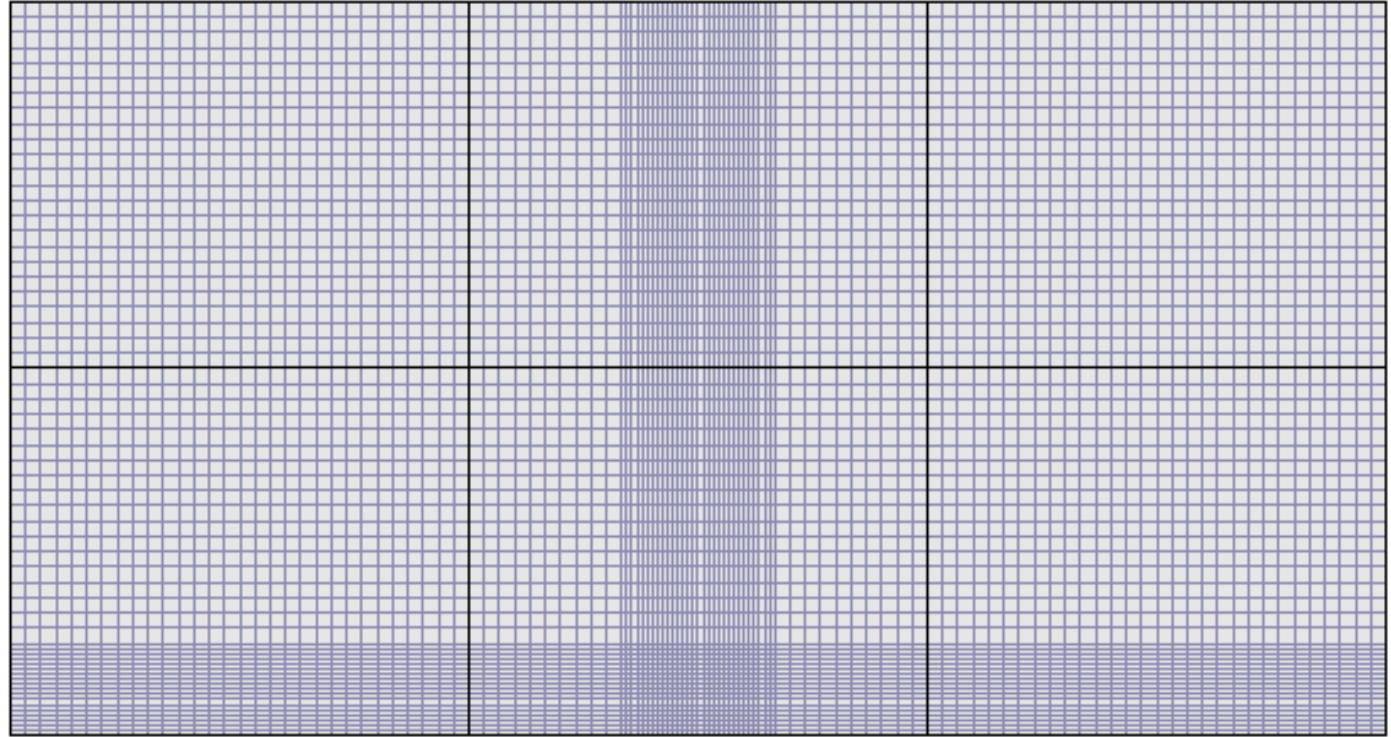
Customize

Nominal Element Size: 4

Generate Mesh OK

Mesh Size Effect

- Coarse mesh (larger elements)
 - Faster solution
 - Smaller output files
 - Less accurate solution
- Finer mesh
 - Slower solution
 - Larger output files
 - More accurate solution



Mesh refinement is not simply about "more elements = better."

Modeling of Constructed Layers

- Thickness
- Modulus of elasticity
- Poisson's ratio
- Coefficient of thermal expansion
- Unit weight

The screenshot shows the 'Layers' dialog box with the following settings:

- Number of layers:** 1
- Layer 1:**
 - Name:** Layer 1
 - Element type:** Plate
 - Thickness:** 10
 - Elastic Modulus:** 4.000e6
 - Poisson Ratio:** 0.15
 - Coef. Therm. Exp.:** 4.40e-6
 - Unit Weight:** 0.087
 - Interface with Layer Above:** Disable for first layer
 - Vertical Stiffness:** (empty)
 - Horizontal Stiffness:** (empty)
 - Friction coefficient:** (empty)
 - Max Friction Stress:** (empty)
 - Description:** (empty)
- Buttons:** Add Layer, Delete Layer, Edit Batch ..., Edit Exceptions ...
- Options:** ☒ Default, ☐ Batch, ☐ Exceptions, ☒ Uniform cross section, ☐ Nonuniform cross section
- Bottom Buttons:** Edit Cross Section, OK

Subgrade Modeling in ISLAB2000

Subgrade

☒ Winkler Subgrade k: Friction model: ▼

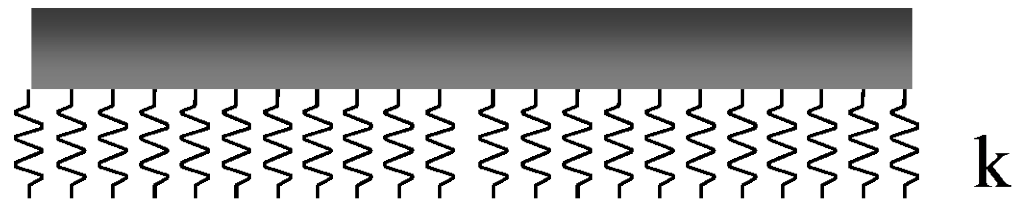
☐ Spring Vlasov and Kerr G:

☐ Vlasov Friction coefficient:

☐ Kerr Kerr Ku: Max Friction:

Description:

☒ Default ☐ Batch ☐ Exceptions



Tensionless foundation



$k = \text{pressure/deflection}$

Modeling of Surface Loads

- Rectangular footprint
- Aspect ratio
- Number of wheels/footprints
- Tire pressure
- Wheel spacing
- Axle coordinates
- Axle weight

Axle Design

Select Axle: New Axle

Axle Name: Forklift

Tire Pressure: 200

Geometry

☒ Aspect Ratio (b/a): 1

☐ Tire Width (b):

Wheel Spacing

☒ S1: 60

☐ S2:

☐ S3:

Axle Spacing

☒ L1: 60

☐ L2:

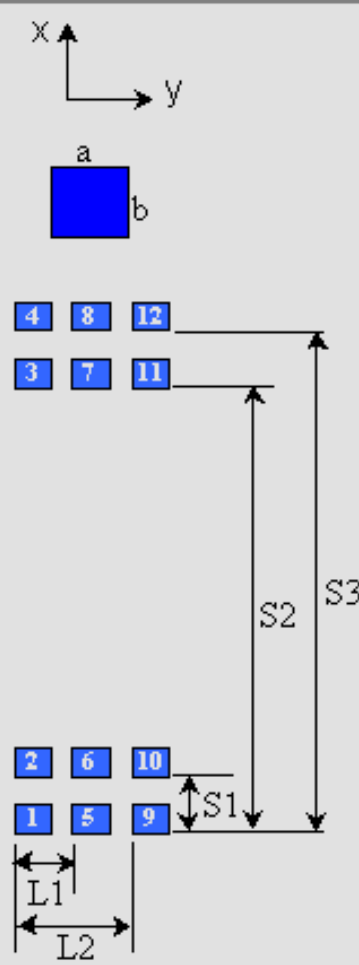
Reference Point

Position: Bottom Left

X-Coordinate:

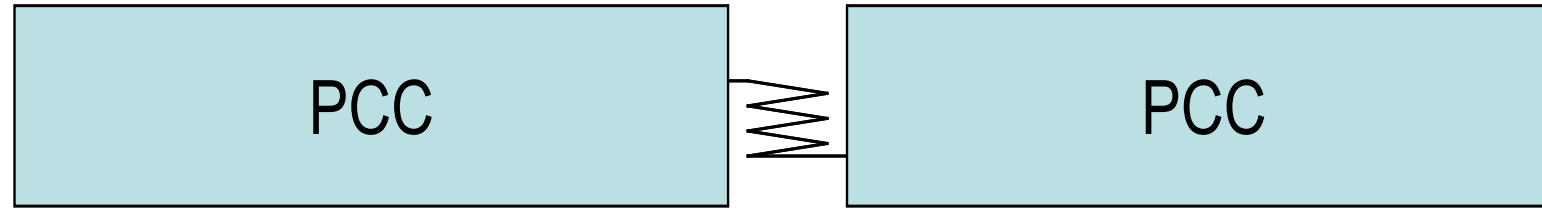
Y-Coordinate:

Delete Clear Save OK

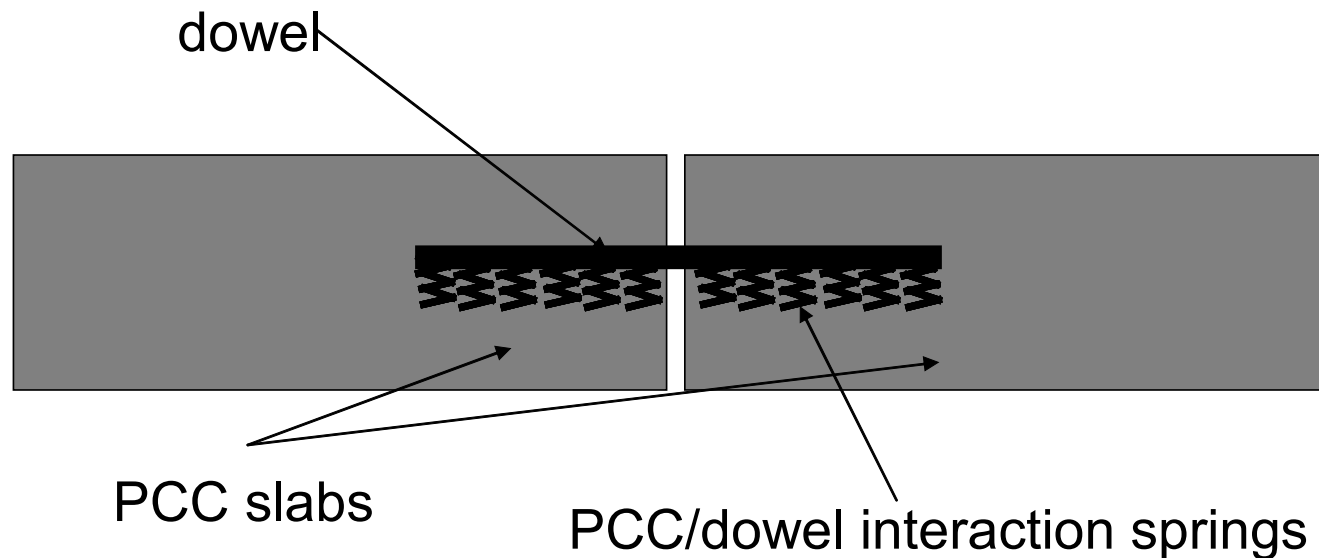


Modeling of Joints

- Aggregate interlock load transfer



- Dowel load transfer



Modeling of Joints

Joints

 **Joints in x-direction**

Number of joints in x-direction: 2

☒ Specify LTE
☐ Specify joint parameters

LTE
Deflection LTE:

Joint parameters

Joint type:

AGG factor:

Normal Stiffness:

Shear Stiffness:

Width:

Crack depth ratio:

Crack Location:

Dowel property ID:

Dowel location ID:

 **Joints in y-direction**

Number of joints in y-direction: 2

☐ Specify LTE
☒ Specify joint parameters

LTE
Deflection LTE:

Joint parameters

Joint type:

AGG factor:

Normal Stiffness:

Shear Stiffness:

Width:

Crack depth ratio:

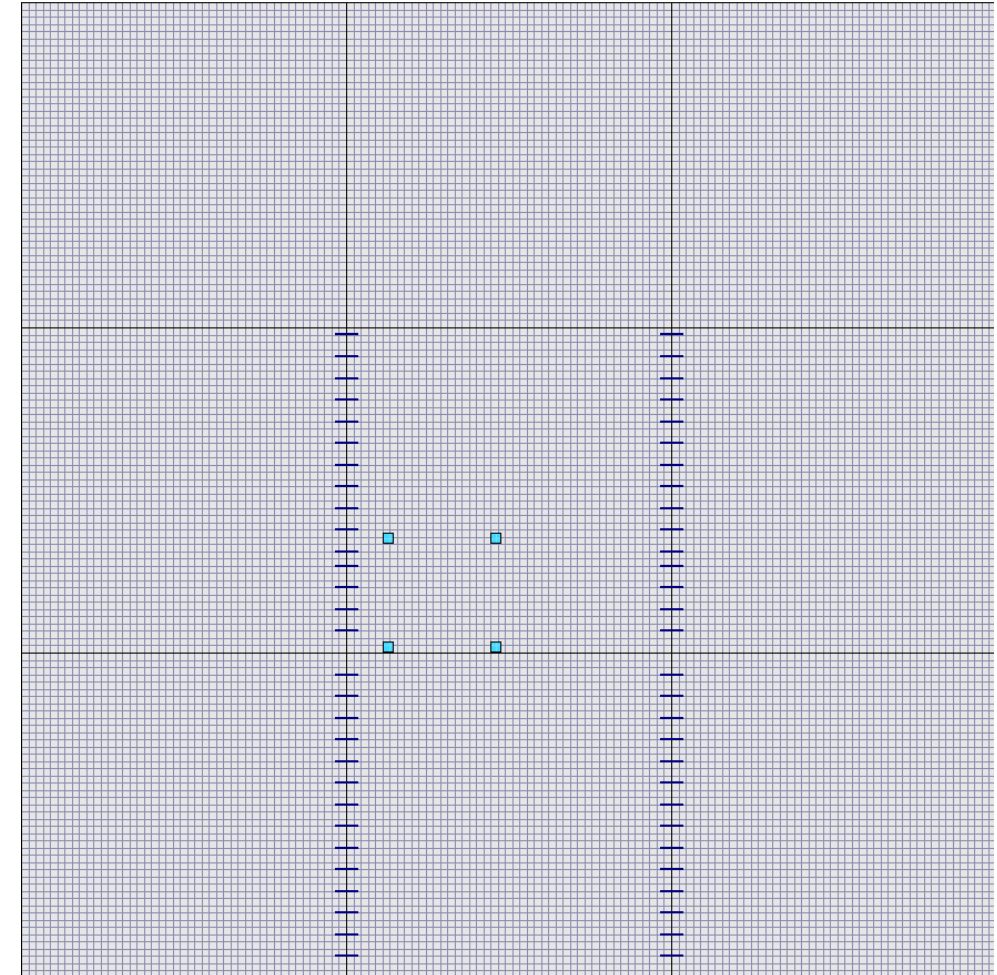
Crack Location:

Dowel property ID:

Dowel location ID:

☐ Exceptions

☐ Batch



Modeling of Curling and Warping

Layers Temperature Properties

☒ Perform Temperature Analysis

Temperatures							
Layer	Type	Difference	Top	Middle	Bottom	Reference	
1	Linear	-30					

☐ Batch

Layers Temperature Properties

☒ Perform Temperature Analysis

Temperatures							
Layer	Type	Difference	Top	Middle	Bottom	Reference	
1	Nonlinear						

☐ Batch

Edit Nonlinear Temperature distributions

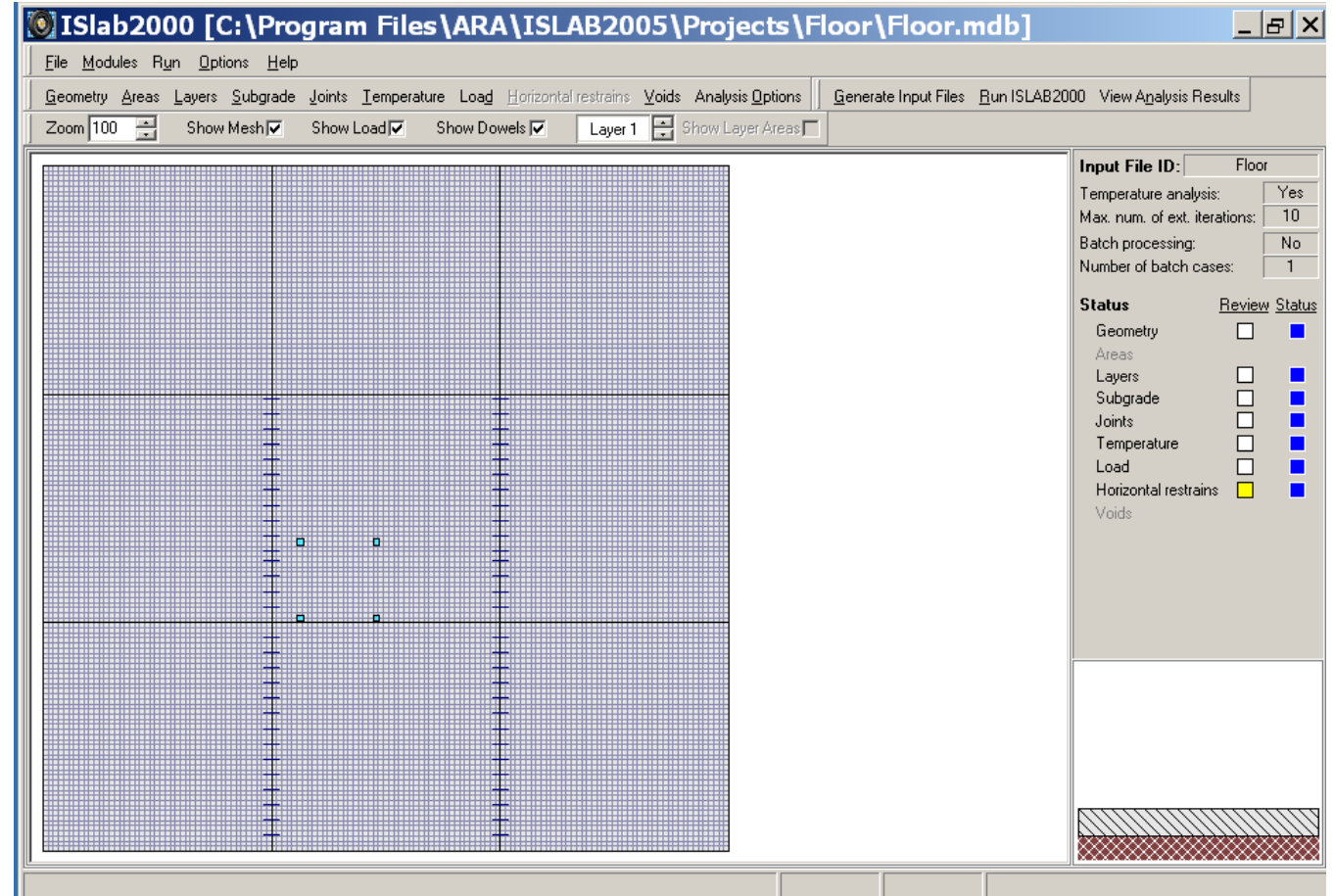
Layer 1

Layer 1: Nonlinear temperature distribution

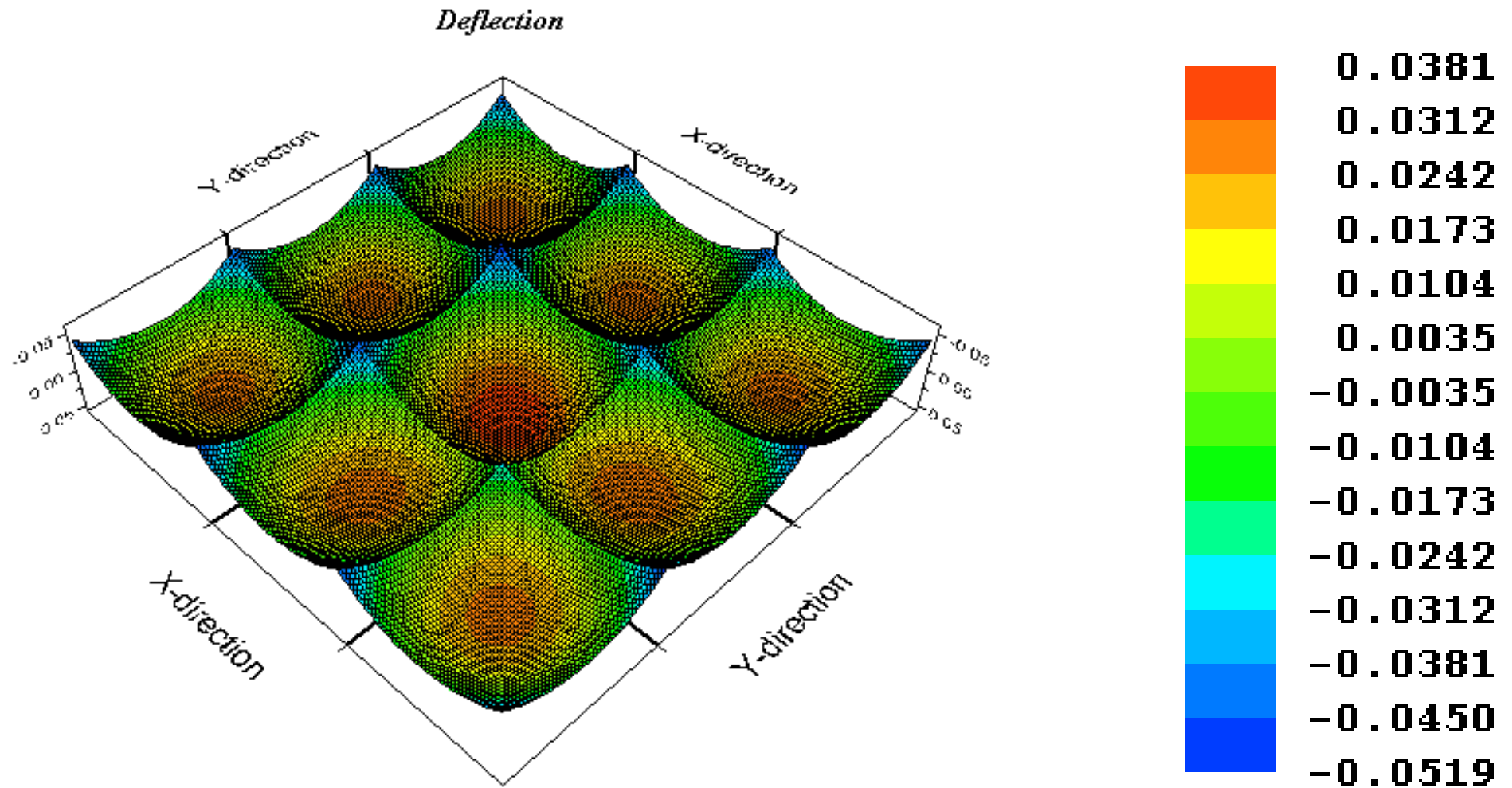
Case	0	3	5	8	10
1	30.00	45.00	50.00	55.00	60.00

ISLAB2000 Analysis Process

- Create model using ISLAB2000Prep
- Run ISLAB2000 analysis
- View results using ISLAB2000Post



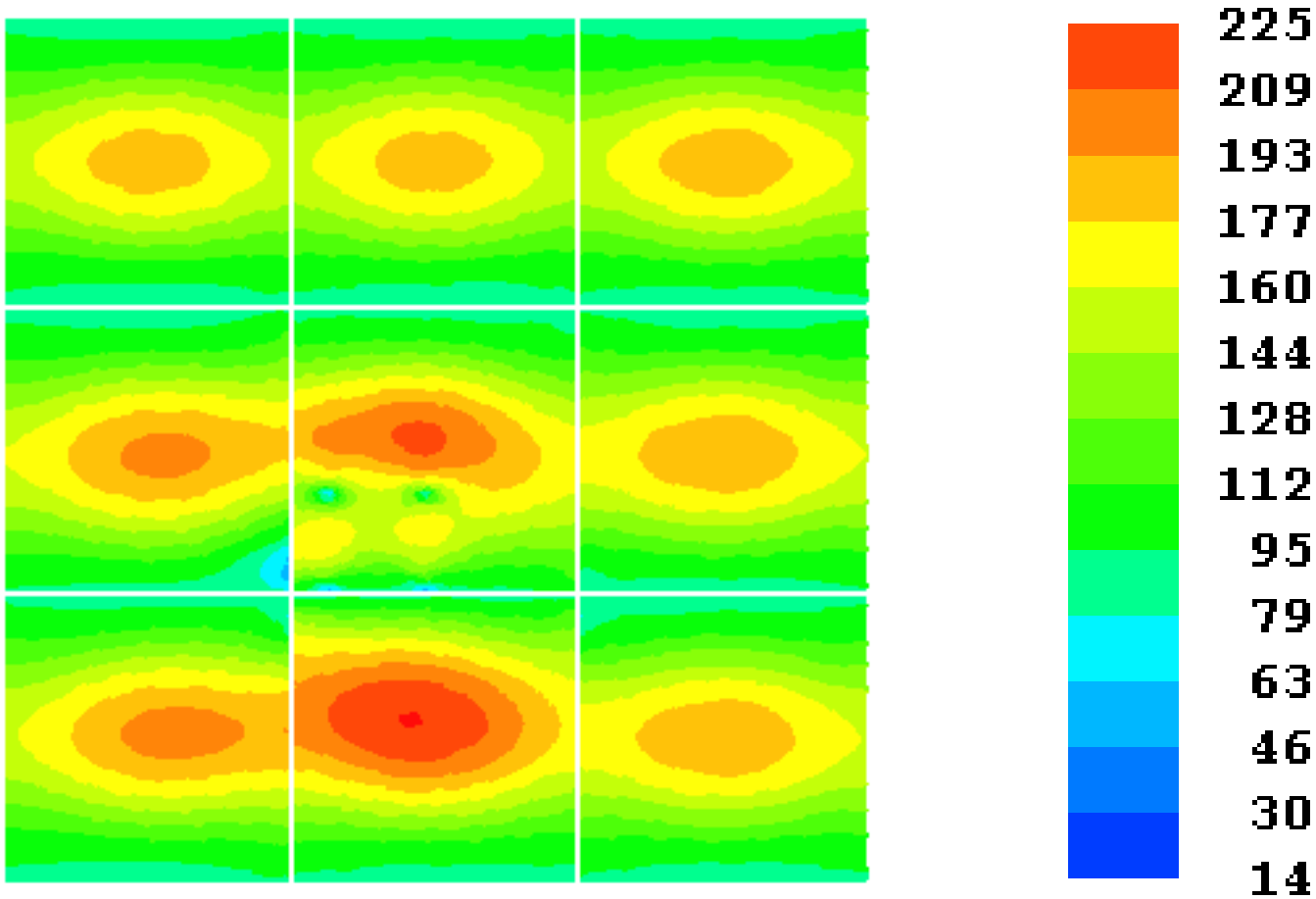
ISLAB2000 Slab Deflections



Westergaard: 0.005 -0.02 in

ISLAB2000 Slab Top Surface Normal Stresses

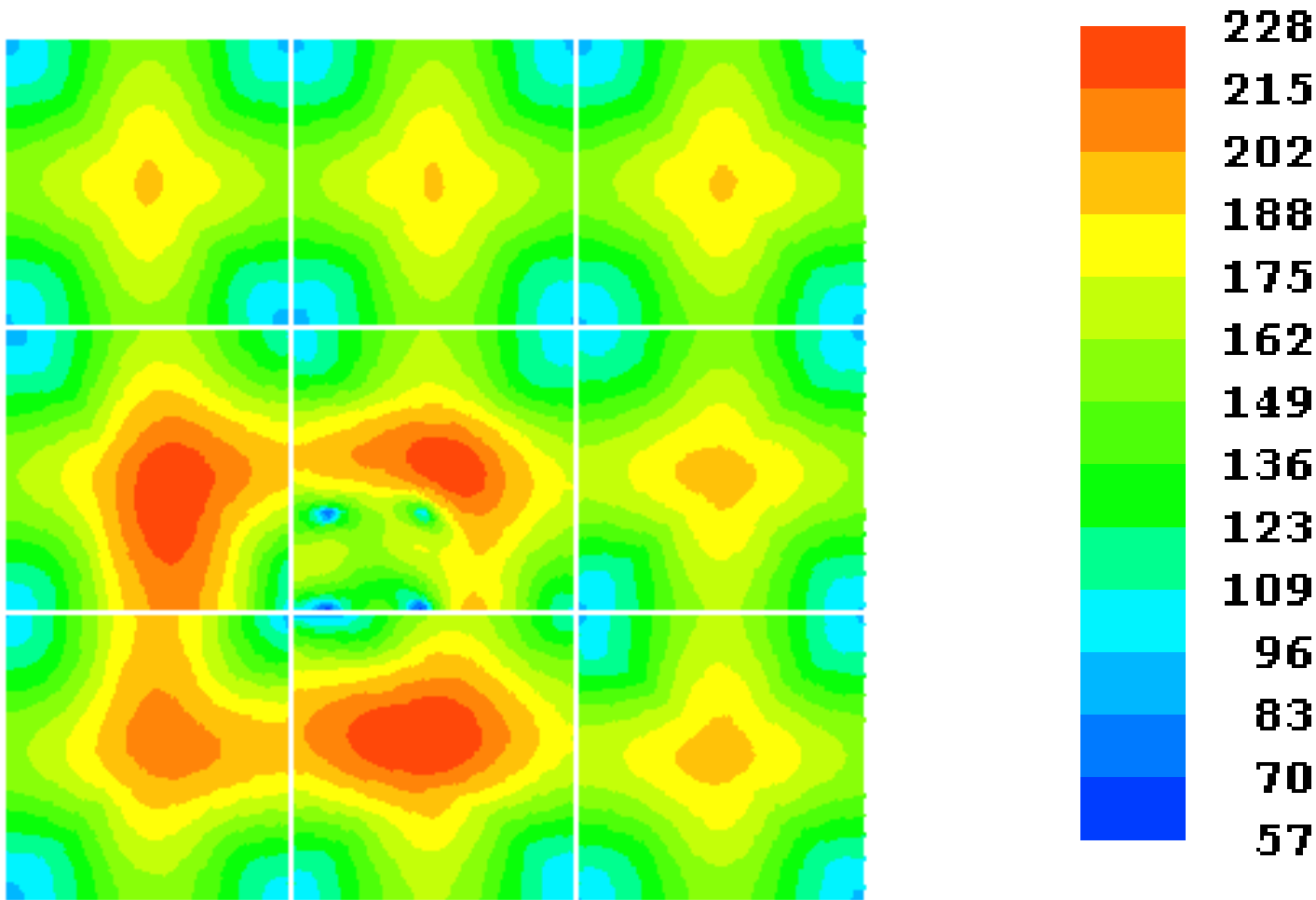
Stresses in X-direction



Westergaard: 270 - 660 psi

ISLAB2000 Slab Top Surface Principal Stresses

Principal Stresses



Why ISLAB is well suited for concrete pavement analysis

- Westergaard solutions are elegant but limited by strong idealizations (single infinite slab, full contact, single wheel, linear temperature, no joints)
- These idealizations can cause significant stress errors under real jointed, multi -axle, and curling conditions
- ISLAB removes the limitations: multi -slab systems, realistic joints, layer separation, multi -axle loads, and nonlinear temperature
- ISLAB was used for the MEPDG development, it supplies the mechanistic accuracy modern design and analysis require
- Use Westergaard for quick estimates; use ISLAB for virtually all real jointed concrete pavement work



Key Takeaways : The question is not whether a model is simple or complex: it is whether its assumptions are consistent with the pavement we are trying to understand .

Thank You

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