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The capacity of a slab is not defined by “thickness”

# Slab Design Webinar



|               |                                                                  |
|---------------|------------------------------------------------------------------|
| Noon – 12.10  | Nigel Parkes   <b>The Need &amp; Introductions</b>               |
| 12.10 – 12.30 | Juan Pablo Covarrubias   <b>The How &amp; Proof of Concept</b>   |
| 12.30 – 12.55 | Dr. Lev Khazanovich   <b>The Numerator</b>                       |
| 12.55 – 1.20  | Dr. Jeff Roesler   <b>The Denominator</b>                        |
| 1.20 – 1.30   | Nigel Parkes   <b>Curl Data Collection</b>                       |
| 1.30 – 2.00   | Robert Rodden   <b>The SLABS™ Software Demo &amp; Work Queue</b> |
| 2.00 – 2.20   | All   <b>Questions &amp; Answers</b>                             |
| 2.20 – 2.30   | Nigel Parkes   <b>Closing Remarks and Next Steps</b>             |

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# The Denominator

## Limitations of Beam Test and Advancements in Slab Capacity Estimation

*Slab Design Workshop  
International Webinar  
August 19, 2026*

**Jeffery Roesler, Ph.D., P.E.**

Ernest J. Barenberg Endowed Professor  
Department of Civil and Environmental Engineering

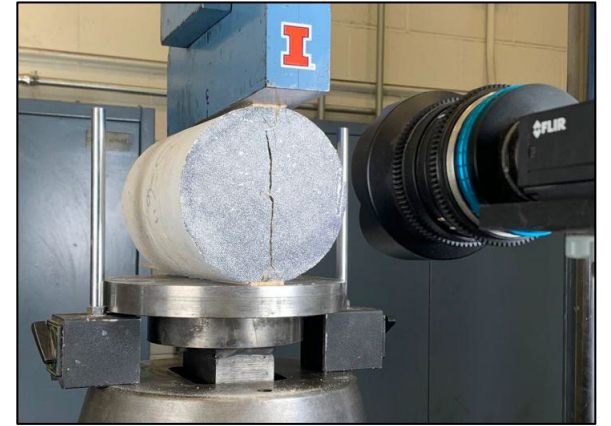


# Question#1

- What concrete strength (R) measure should we use for concrete slab design?

$$\text{Stress ratio} = \frac{\sigma}{R}$$

- $\sigma$  = Slab Tensile Stress
- R = Concrete Strength



?

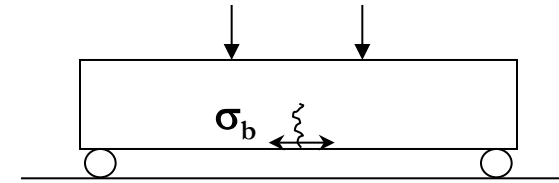
# Concrete “Strength” Statement

## □ Observations: Concrete Slab “strength” greater than beam strength

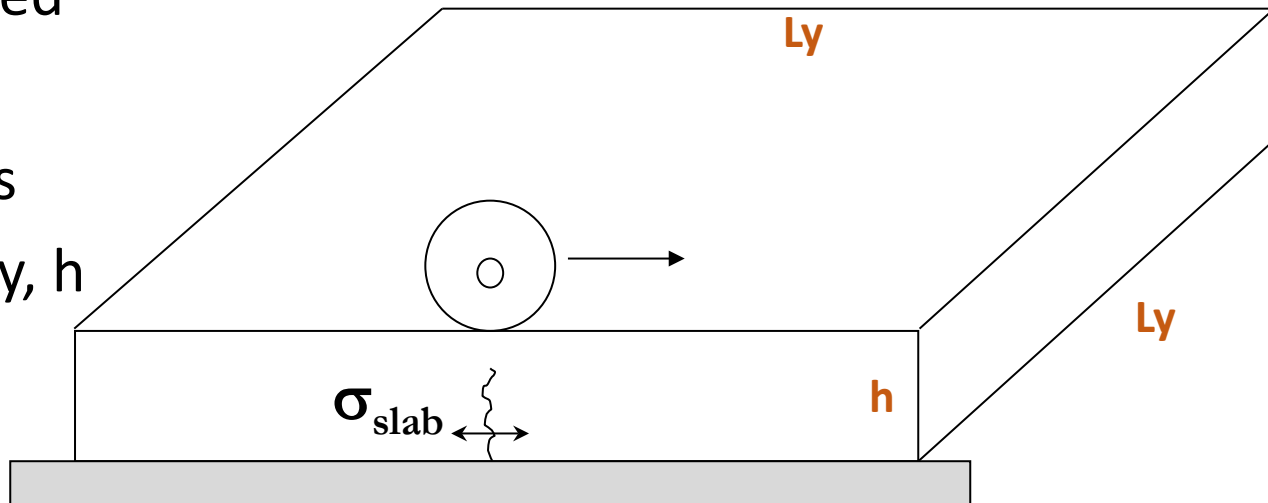
□ Designs assume  $\sigma_b = \text{MOR} = \sigma_{\text{slab}}$

## □ Strength difference exists:

- Incorrect definition of strength
- Other mat'l properties ignored
  - $G_c$ - fracture toughness
- Loading configuration effects
- Slab geometry effects –  $L_x$ ,  $L_y$ ,  $h$
- Boundary condition effects



$$\sigma_b \neq \sigma_{\text{slab}}$$



$$\sigma_b = \text{MOR} = \text{flexural strength}$$

# Presentation Objectives:

## Defining Concrete Strength for Design

1. What do we currently use ( $\sigma_{\max} / R$ ) in the denominator for strength ( $R$ ) and what does it actually measure?
2. What should we use in the denominator for strength ( $R$ ), e.g., MOR,  $\sigma_{\text{split}}$ ,  $\sigma_{\text{cs}}$ ,  $\sigma_{\text{ts}}$ ?
3. What are necessary concrete “strength” inputs for slab design?
4. *How do we determine slab capacity (slab strength)?*

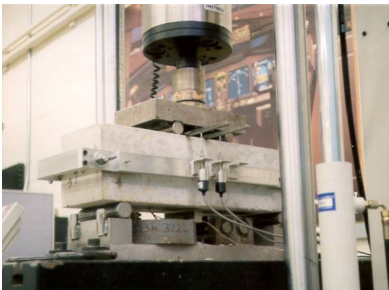
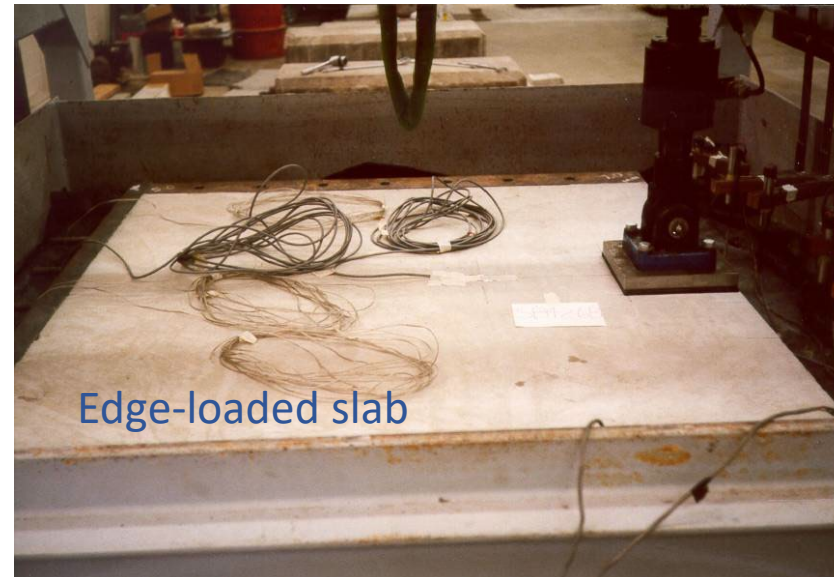
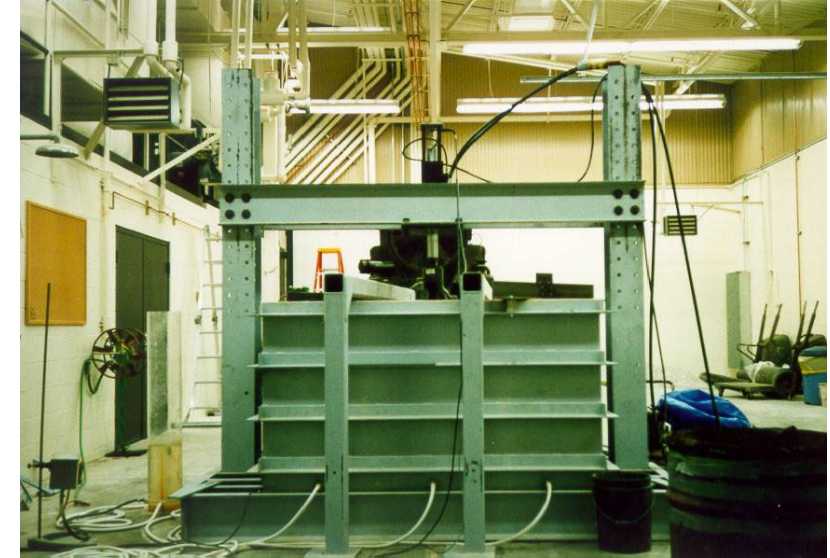
# Outline for Presentation

- Slab tests in laboratory
  - Plain and fiber reinforced concrete
- Full-scale slab tests - field
- Flexural beam test (MOR) re-interpretation
- Goal - Slab Capacity

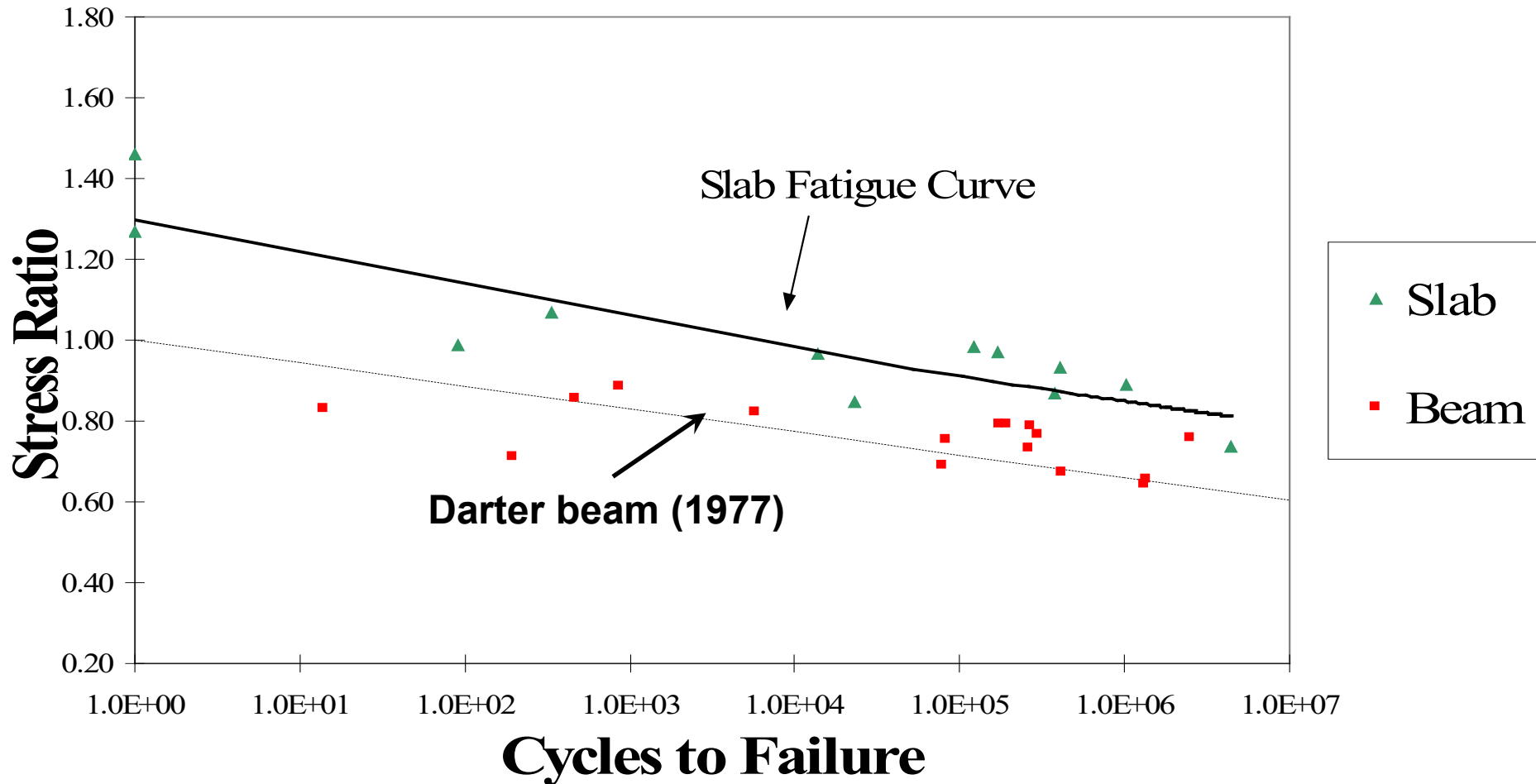
# Monotonic and Fatigue of Concrete Slabs

## Fully Supported Slabs

- 1.22m x 1.22m x 0.15m
- 10 slabs cyclically loaded
- 2 slabs statically tested
- 24 beams for flexural strength



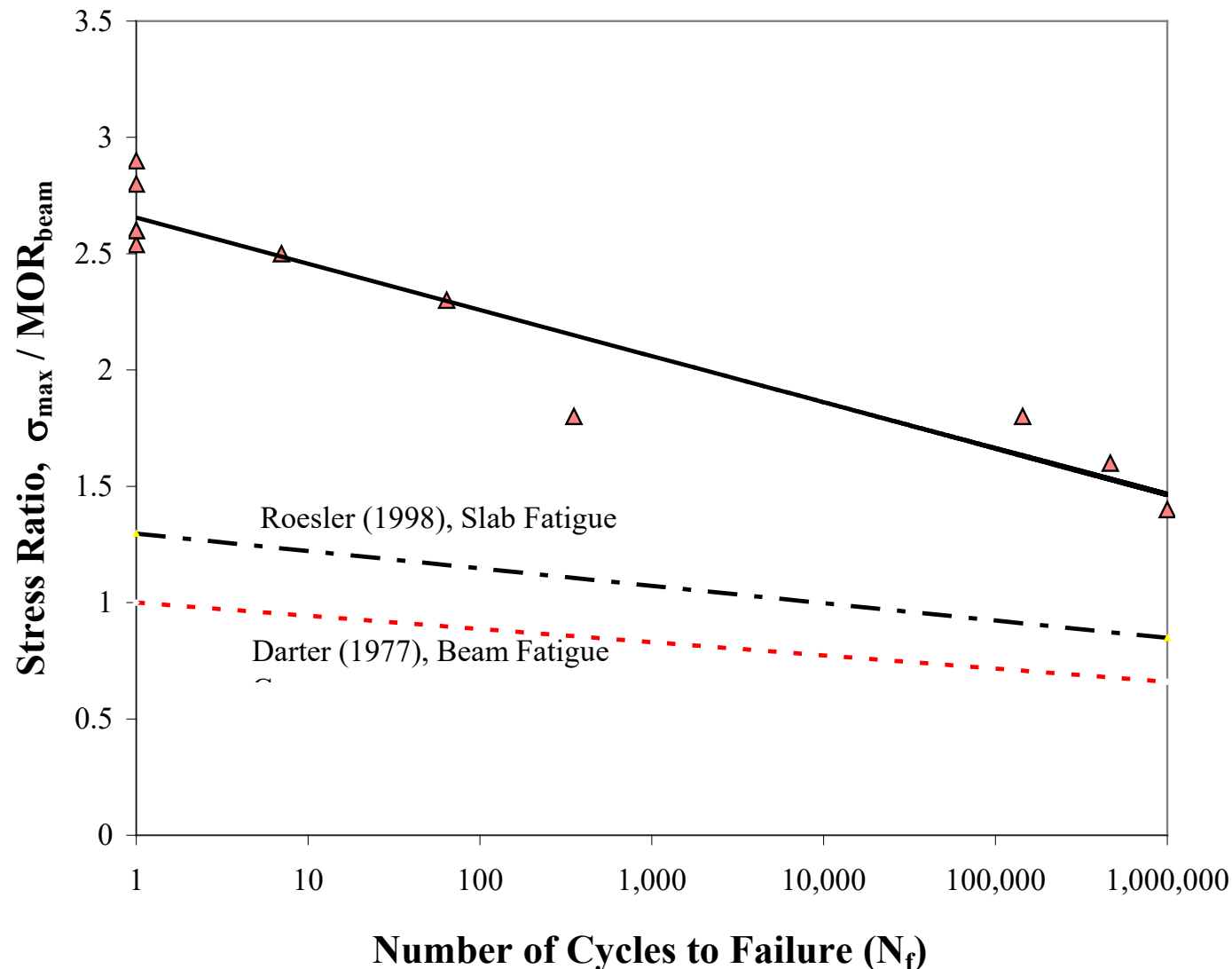
# Beam and Slab Fatigue Curves



$$N_f = \left[ \frac{1.2968}{(\sigma / \text{MOR}_{\text{beam}})} \right]^{32.57}$$

*Q#2 - Is beam to slab strength fixed or variable?*

# Concrete Slab Test Results - Single Pulse

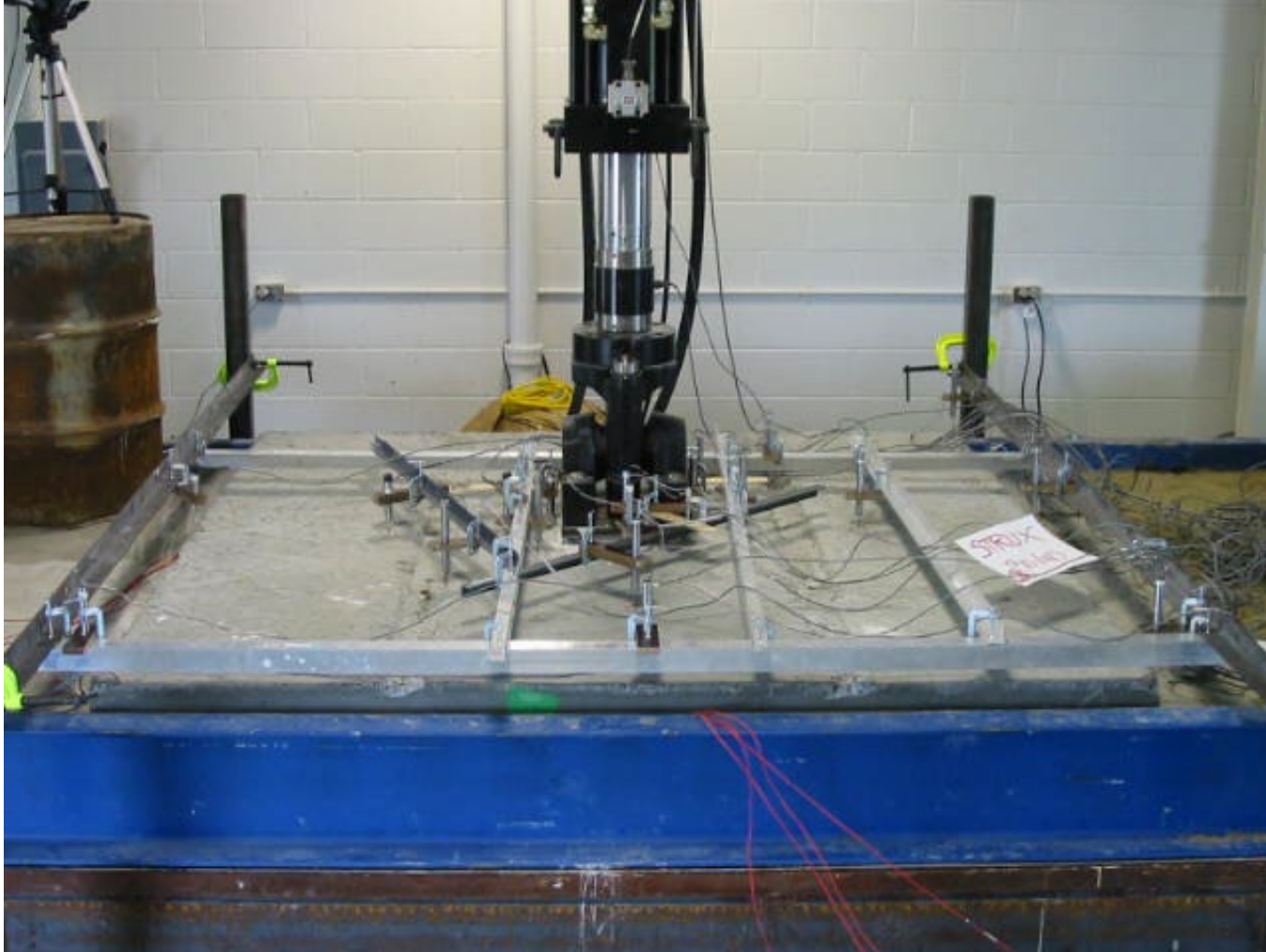


$$N_f = \left[ \frac{2.689}{(\sigma / \text{MOR}_{\text{beam}})} \right]^{21.79}$$

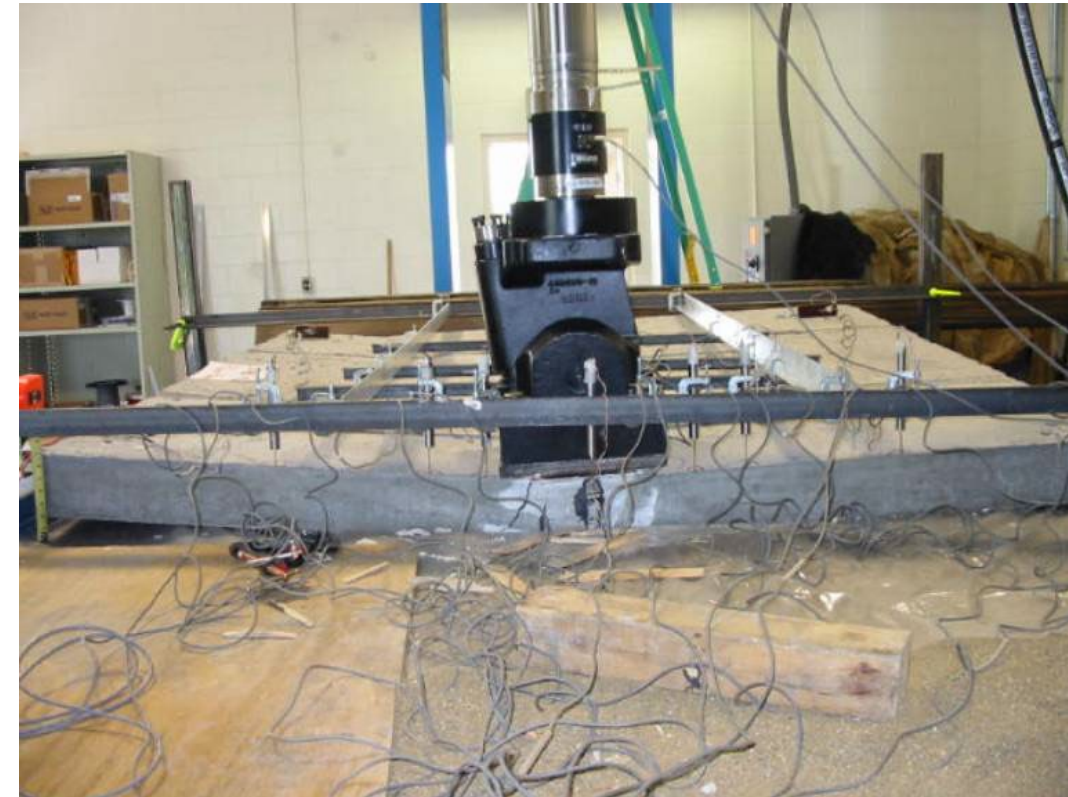
• Slab strength = 2.7 \* MOR

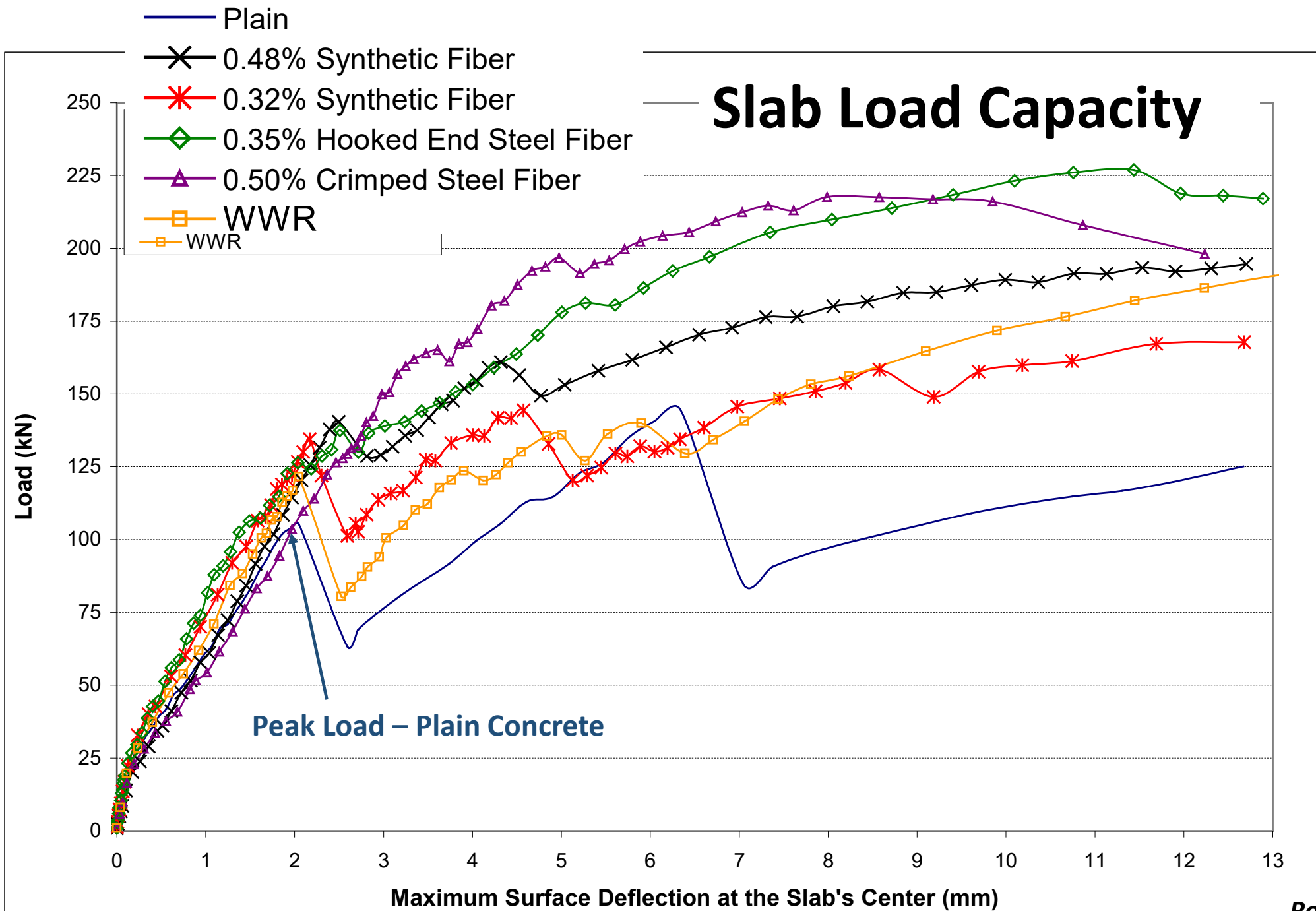
# Monotonic FRC Slab Testing (macrofibers)

Interior Loading



Edge Loading





# Slab Failure Load Summary (kN)

*Roesler et al. (ASCE 2004)*

| Slab Type               | Tensile | Flexural | Ultimate | Slab/Beam<br>Flex. Strength |
|-------------------------|---------|----------|----------|-----------------------------|
| Plain                   | 75      | 108      | 135      | 1.39                        |
| STRUX 90/40<br>(0.48%)  | 70      | 143      | 195      | 2.09                        |
| STRUX 90/40<br>(0.32%)  | 75      | 135      | 174      | 1.82                        |
| Dramix 65/60<br>(0.35%) | 70      | 141      | 228      | 2.01                        |
| Tabix 1.3/50<br>(0.50%) | 90      | 167      | 220      | 2.22                        |
| WWR                     | 65      | 122      | 201      | 1.53                        |

# Summary of Concrete Slab Testing (Abridged)

| Reference                    | Concrete Materials         | Slab dimensions       | Monotonic / Fatigue | Load Location     | Slab to beam Strength       |
|------------------------------|----------------------------|-----------------------|---------------------|-------------------|-----------------------------|
| Roesler (1998)               | Plain Concrete             | 1.22 x 1.22 x 0.152 m | Both                | Edge              | 1.3                         |
| Roesler et al. (2004a)       | Plain Concrete             | 2.0x2.0x 0.15m        | Both                | Edge              | 2.8                         |
| Kohler (2005)                | CRCP <sup>1</sup>          | Variable              | Fatigue             | Edge              | >1.0                        |
| Rao (2005)                   | FSHCC <sup>2</sup>         | Variable              | Fatigue             | Edge              | 2.3                         |
| Roesler et al. (2004b, 2006) | Plain and FRC <sup>3</sup> | 2.2x2.2x 0.125m       | Monotonic           | Edge and interior | 1.4 Plain; 1.8 to 2.2 (FRC) |
| Beckett and Humphreys (1989) | FRC and plain              | 3.0 x3.0 x 0.15m      | Monotonic           | Interior          | 2.0 Plain; 2.2 to 3.2 (FRC) |
| Falkner and Teutsch (1993)   | FRC and plain              | 3.0 x3.0 x 0.15m      | Monotonic           | Interior          | 2.1                         |
| Jensen and Hansen (2001)     | Plain Concrete             | 3.0 x 1.8 x 0.25m     | Monotonic           | Uniaxial tension  | N/A                         |

*Roesler (2006)*

# 9cm and 15cm Concrete Slab Tests (2010)

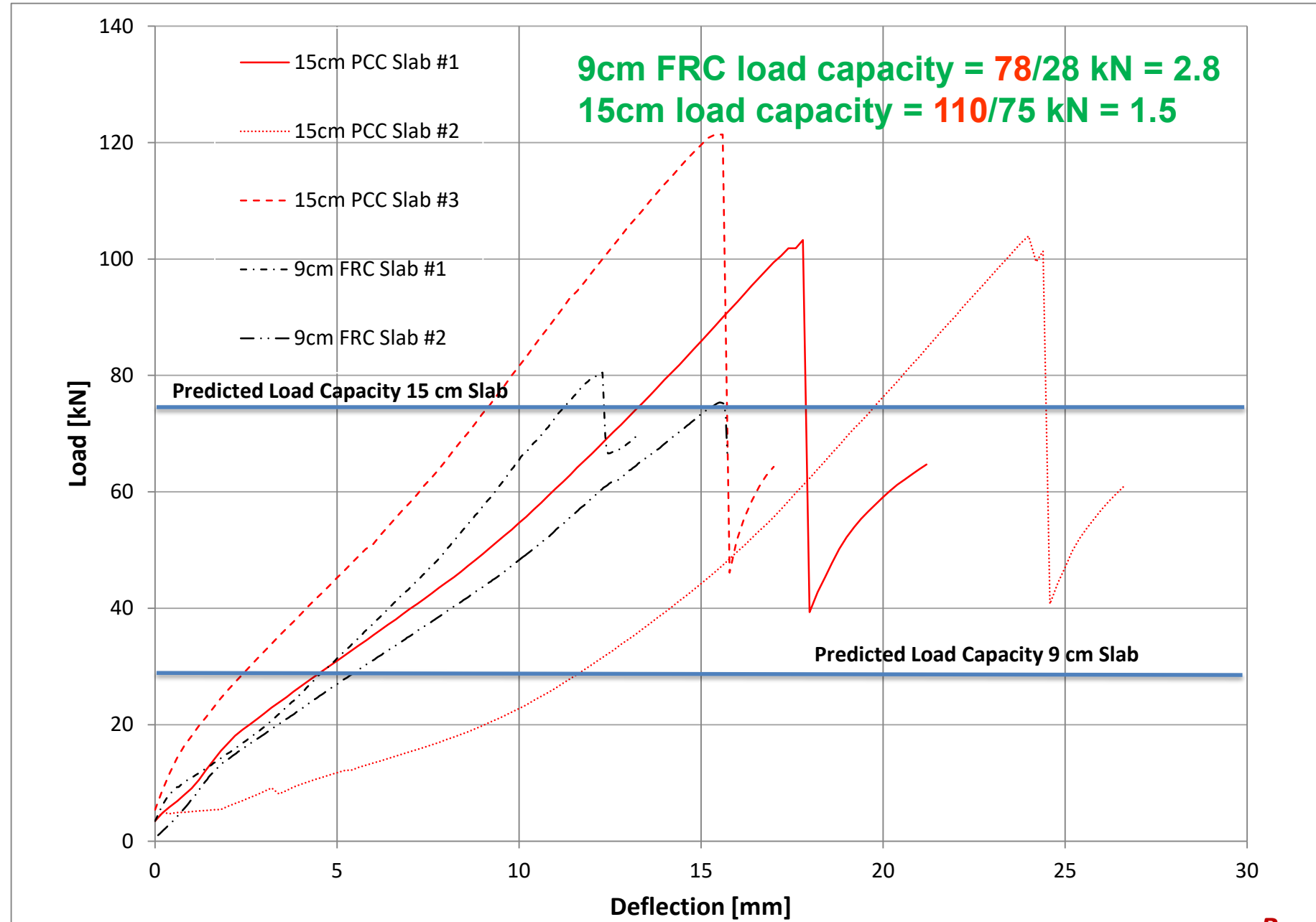


1.8m x1.8m slab size by 9cm thick



Edge loading: 9cm (FRC) or 15 cm (plain) thickness

# Actual vs. Predicted Slab Capacity



# Short Jointed Slab Fatigue Design

$$\log(N_{ijkl}) = 2 \times \left( \frac{C_3 \times \sigma_{ijkl}}{MOR \times C_1 \times C_2} \right)^{-1,22}$$

**$C_1$  = Slab Geometry & Thickness factor**

**$C_2$  = macro-fiber correction factor**

**$C_3$  = *Edge loading correction factor***

$N_{ijkl}$  = allowable passes for axle position k, curling i (temperature), load level j, and critical top or bottom stress location /

$\sigma_{ijkl}$  = principal stresses calculated with ISLAB2000 for axle position k, curling i and load j, and critical top or bottom stress location /

MOR = Flexural strength of concrete at 90 days

# What does the concrete beam test measure?

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- Third-point loading:

$$S_{max} = \frac{P_{max}L}{DH^2}$$

- $S_{max}$  (MOR) = ?
- $P_{max}$  = peak load
- $L$  = span
- $H$  = beam height or depth
- $D$  = beam width

*Q#4 - Is it a fundamental material property?*



# How long have we been studying flexural strength (MOR) for concrete?

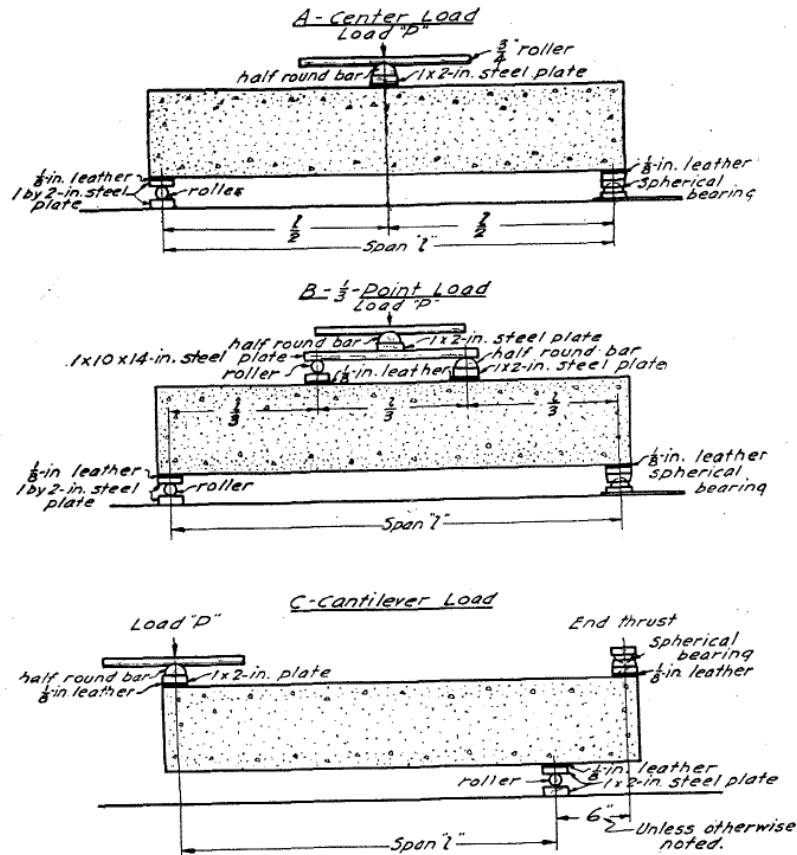


Fig. 16—Methods of Loading Concrete Beams.

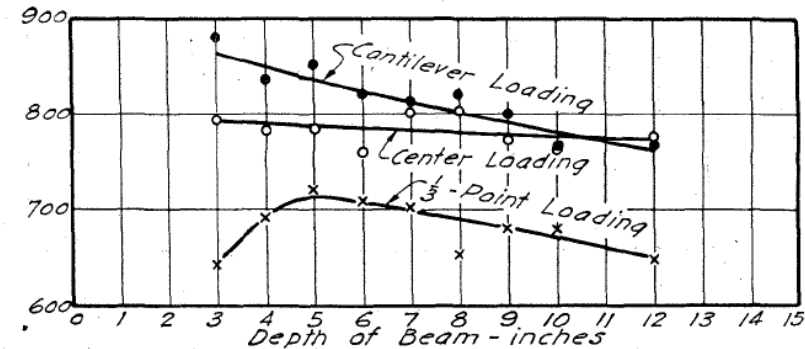


Fig. 17—Relation between Modulus of Rupture and Depth for Constant Width of Beam.

Mix 1-4 by volume. Age at Test, 28 days. Data from Table 1.

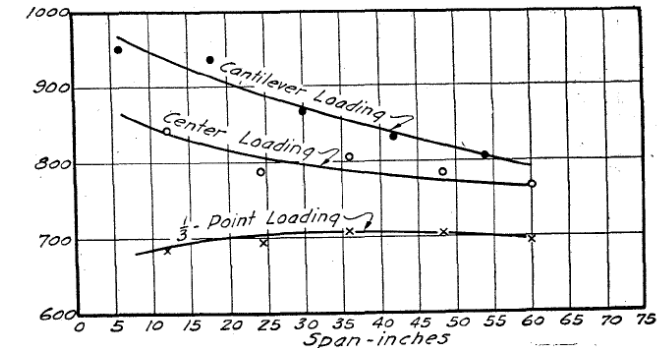
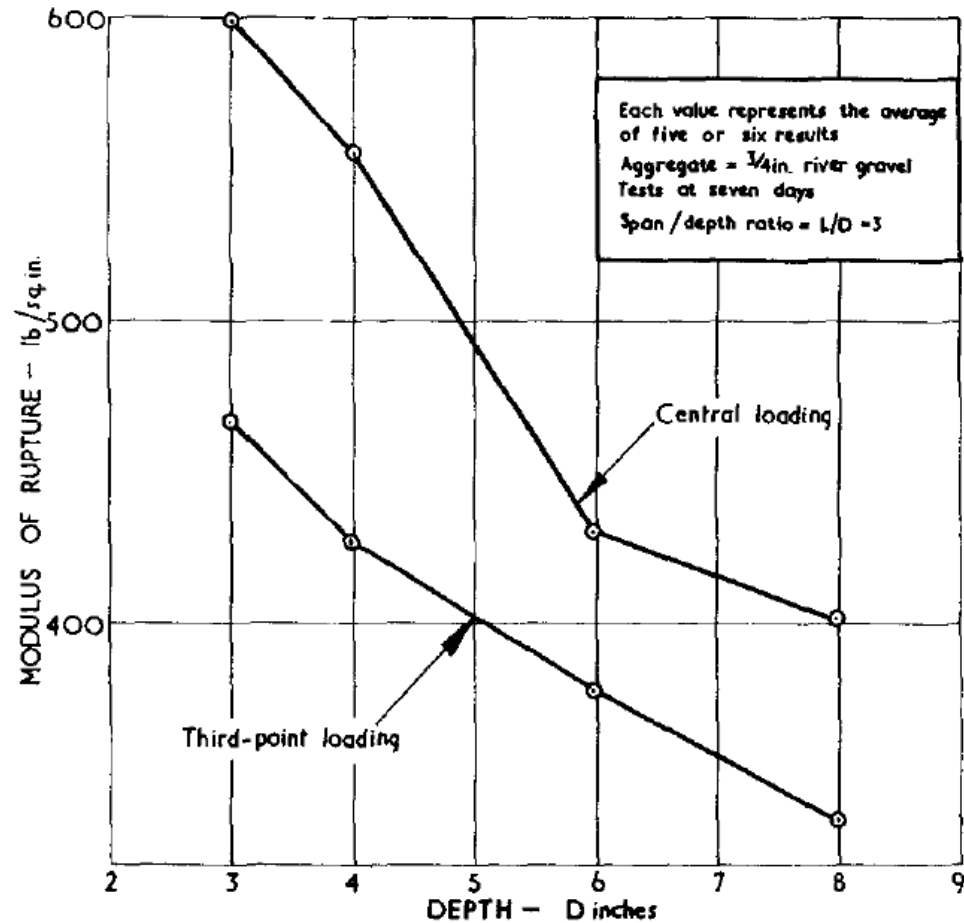


Fig. 20—Relation between Modulus of Rupture and Span.  
Mix 1-4 by volume. Age at Test, 28 days. Data from Table 1.

(7) The tests show that 1/3-point loading has some important advantages over center and cantilever loading for determining the modulus of rupture of concrete. If, on account of ease of making tests, the cantilever method is preferred, it should be more thoroughly standardized than at present so that tests will be carried out under uniform conditions as to a rate and method of loading,

| Curing                                                     | Coarse Aggregate | Relative Strengths of Concrete (From curves Fig. 7)—% |      |     |     |                    |      |     |     |
|------------------------------------------------------------|------------------|-------------------------------------------------------|------|-----|-----|--------------------|------|-----|-----|
|                                                            |                  | Compressive Strength                                  |      |     |     | Modulus of Rupture |      |     |     |
|                                                            |                  | 7d.                                                   | 28d. | 3m. | 1y. | 7d.                | 28d. | 3m. | 1y. |
| Under wet burlap; tested damp                              |                  | 100                                                   | 100  | 100 | 100 | 100                | 100  | 100 | 100 |
| Air of Laboratory; tested dry                              | Gravel           | 90                                                    | 84   | 75  | 65  | 84                 | 77   | 78  | 75  |
|                                                            | Limestone        | 101                                                   | 85   | 80  | 62  | 95                 | 86   | 90  | 87  |
| Air of Laboratory; tested wet                              | Gravel           | 91                                                    | 65   | 54  | 49  | 101                | 110  | 96  | 81  |
|                                                            | Limestone        | 102                                                   | 62   | 61  | 53  | 110                | 110  | 107 | 98  |
| Air of Laboratory; lower half wet                          | Gravel           | 86                                                    | 64   | 53  | 51  | 104                | 100  | 82  | 70  |
|                                                            | Limestone        | 101                                                   | 68   | 62  | 53  | 97                 | 83   | 86  | 76  |
| Air of Laboratory; paraffin coated; tested as cured        | Gravel           | 88                                                    | 97   | 87  | 81  | 108                | 102  | 93  | 91  |
|                                                            | Limestone        | 101                                                   | 93   | 97  | 80  | 119                | 124  | 120 | 103 |
| Air of Laboratory; sodium silicate coated; tested as cured | Gravel           | 98                                                    | 89   | 76  | 65  | 101                | 90   | 87  | 76  |
|                                                            | Limestone        | 104                                                   | 75   | 72  | 61  | 91                 | 85   | 84  | 87  |

# Other Historical Beam Studies



Wright (1952)

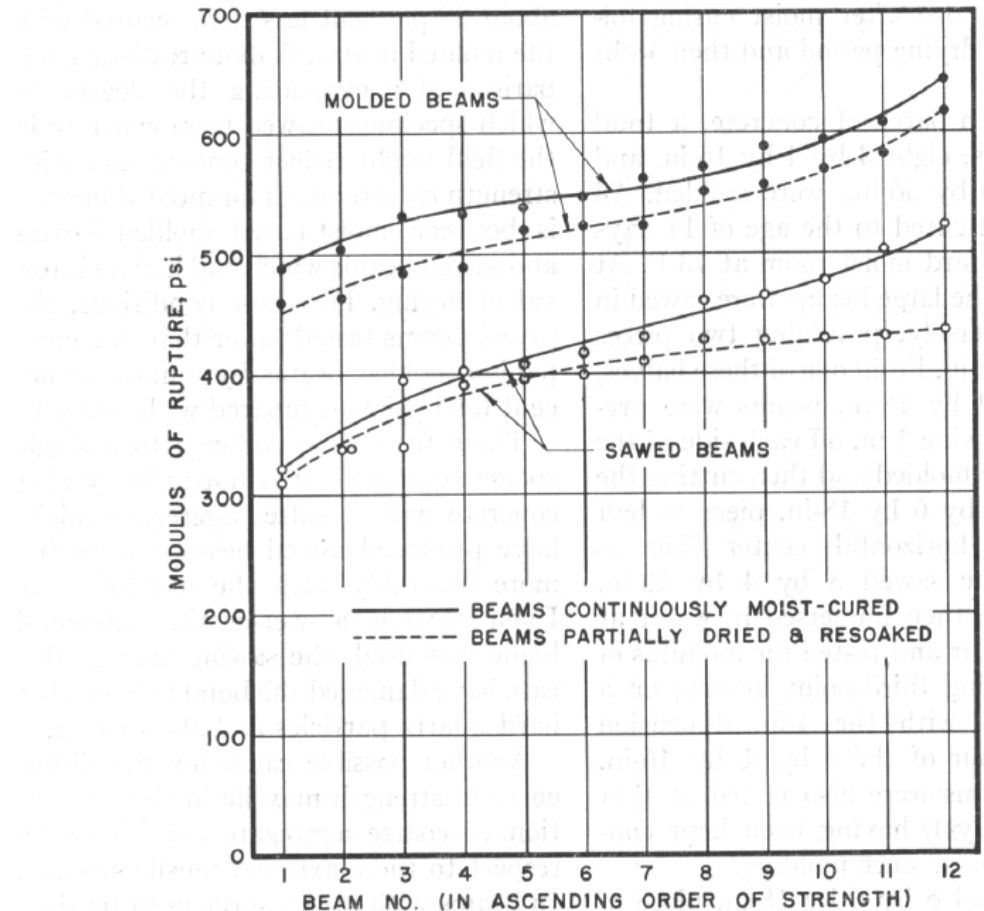


FIG. 4.—Relationship Between Flexural Strengths of Sawed and Molded Beams of Same Concrete (Series 155 E).

Walker and Bloem (1957)

# What assumptions have been made incorrectly about beam flexural strength test?

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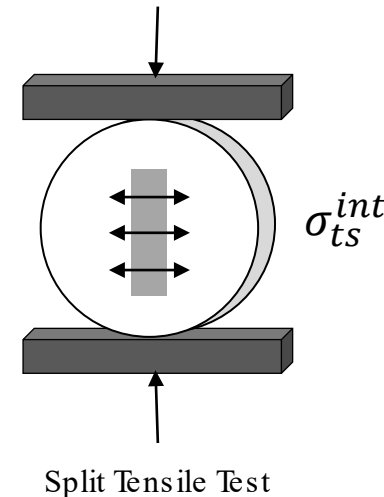
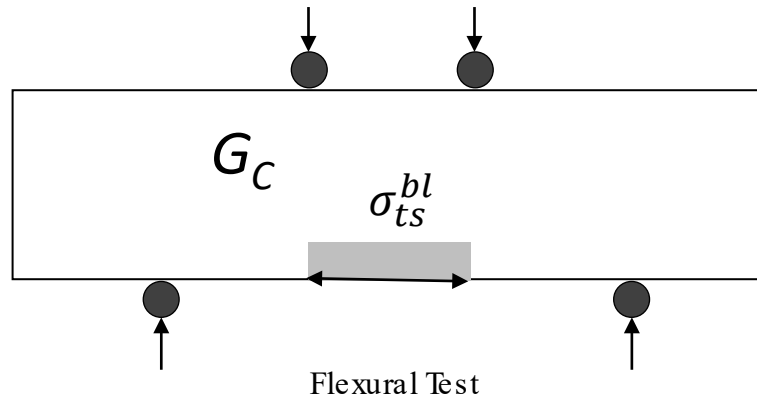
1. *Concrete beam fails when the tensile stress in beam is reached at a critical point/location, e.g., bottom of the beam in the middle third.*
  - No, beams fail when a certain region of the beam exceeds the strength surface  $[F(\sigma)]$ ; and then nucleates & propagates a crack.

$$F(\sigma) = \sqrt{J_2} + \frac{\sigma_{cs} - \sigma_{ts}}{\sqrt{3}(\sigma_{cs} + \sigma_{ts})} I_1 - \frac{2\sigma_{cs}\sigma_{ts}}{\sqrt{3}(\sigma_{cs} + \sigma_{ts})} = 0, \dots$$

2. *Tensile strength of concrete changes with height of the specimen*
  - No, “flexural strength” reduces with ↓ beam height, but the **tensile strength** is the same for any specimen height.
3. *Concrete slabs fail when the applied tensile stress reaches the beam “flexural strength”*
  - No, same as (1) above

# What does beam test measure?

- Flexural strength (MOR) combines ***concrete tensile strength,  $G_c$ , loading configuration, and specimen geometry*** (*structural strength*)
- Tensile strength near the (bottom) boundary layer of the specimen affects the flexural strength.
- Not true for Brazilian split tensile test – interior of the specimen

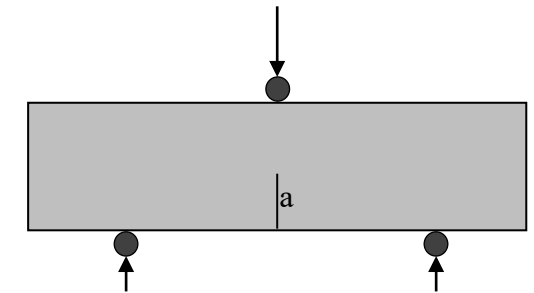


$\sigma_{ts}^{bl}$  = tensile strength of boundary layer  
 $\sigma_{ts}^{Int}$  = tensile strength of interior

$\sigma_{ts}^{bl} = \text{or } \neq \sigma_{ts}^{Int}$

# What is $G_c$ for concrete materials?

- Critical strain energy release rate or *fracture toughness* (Griffith) -  $G_c$
- Material fracture property
  - minimum energy required to create new material surface
- How do we measure it → Notched specimen tests
- $G_c$  value – 10 to 50 N/m
  - Depends on w/c, aggregates, ITZ, air voids, etc.
  - Toughening mechanisms – fibers

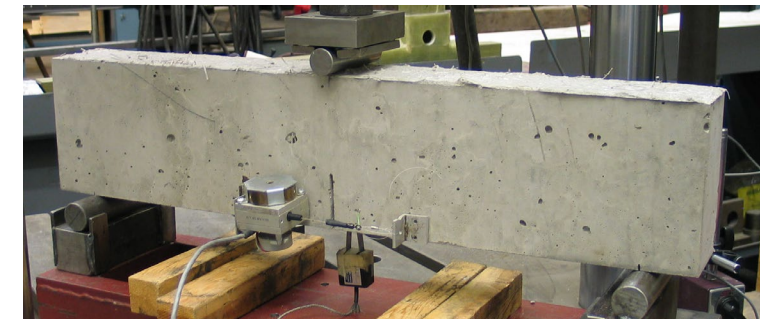


Single Edge Notched Beam

$$\begin{cases} \text{Div} [v_k^2 \sigma(\mathbf{X}, t_k)] = 0, & \mathbf{X} \in \Omega \\ \text{Div} [\varepsilon \delta^\varepsilon G_c \nabla v_k] = \frac{8}{3} v_k W(\mathbf{E}(\mathbf{u}_k)) + \\ \frac{4}{3} c_e(\mathbf{X}, t_k) - \frac{\delta^\varepsilon G_c}{2\varepsilon} + \frac{8}{3\zeta} p(v_{k-1}, v_k), & \mathbf{X} \in \Omega \end{cases}$$

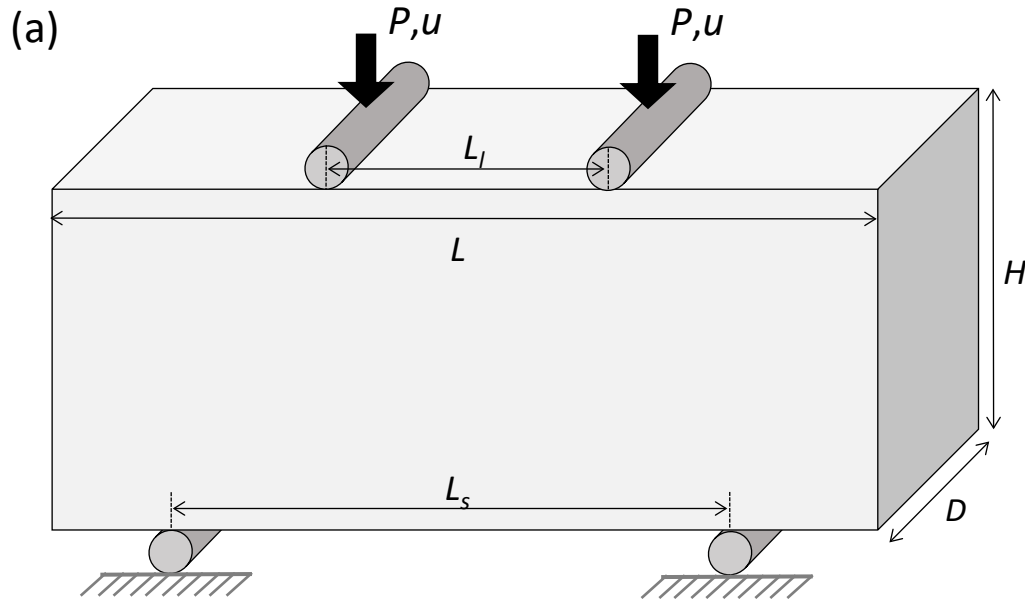


Fig. 2. WST specimen setup



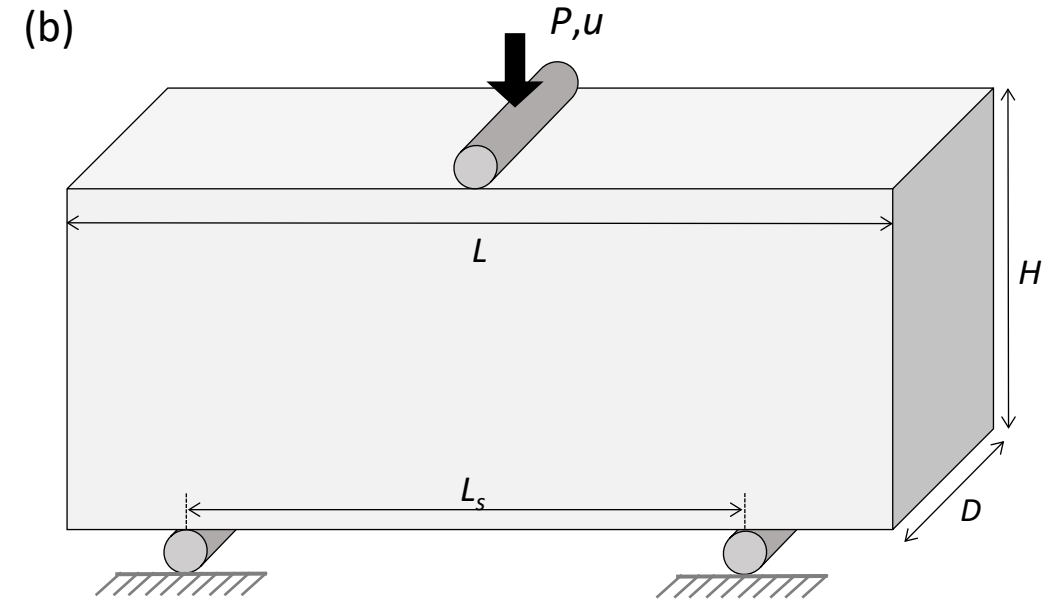
# Is there a theoretical difference in flexural strength?

## 4-Point Bend Test



$$\begin{aligned}L_s/H &= 4 \\L_l/H &= 2 \\D = H &= 1 \\L &= 1.0 \text{ m} \\H &= 0.2 \text{ m}\end{aligned}$$

## 3-Point Bend Test

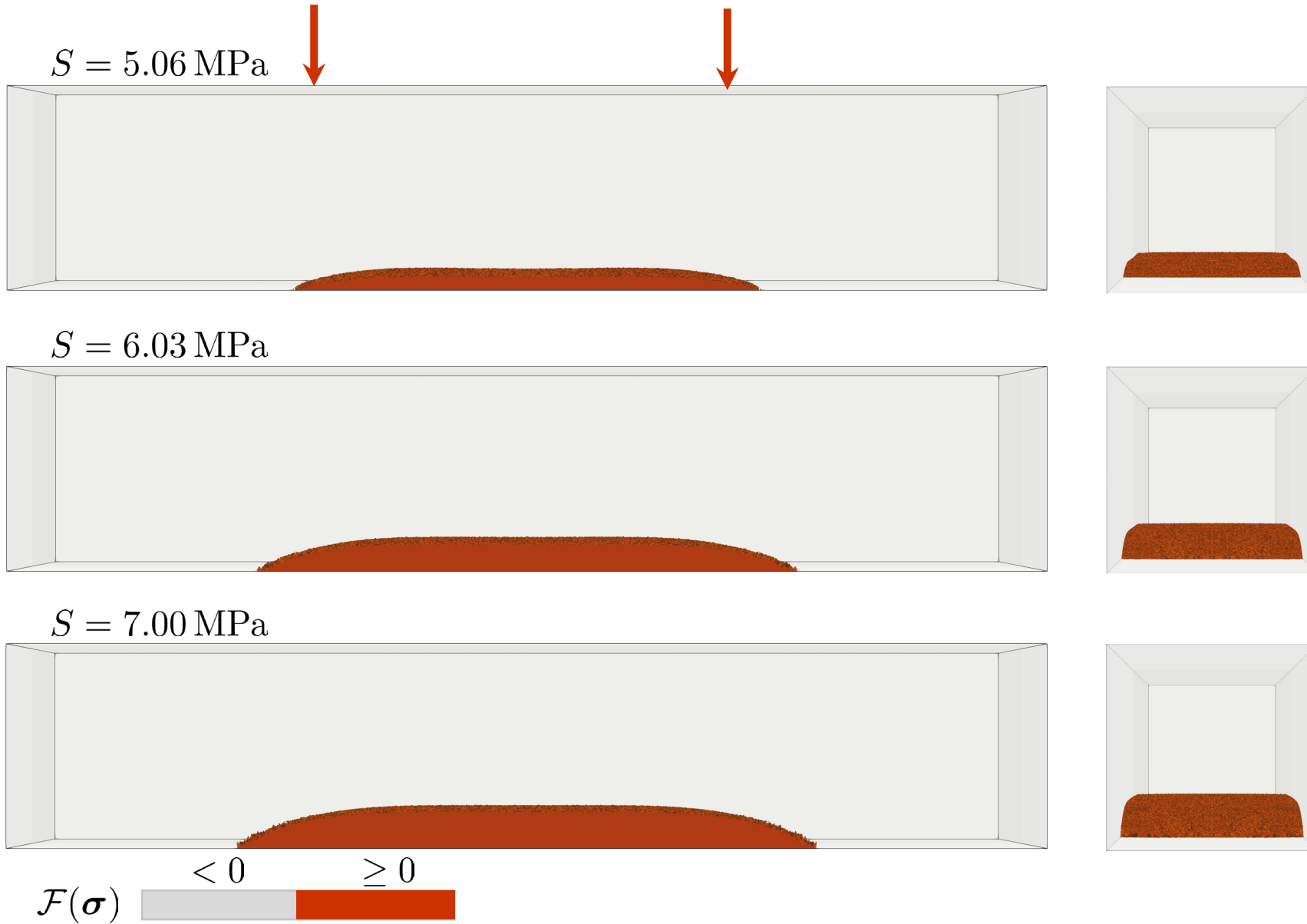


$$\begin{aligned}L_s/H &= 4 \\L_l/H &= 0 \\D = H &= 1 \\L &= 1.0 \text{ m} \\H &= 0.2 \text{ m}\end{aligned}$$

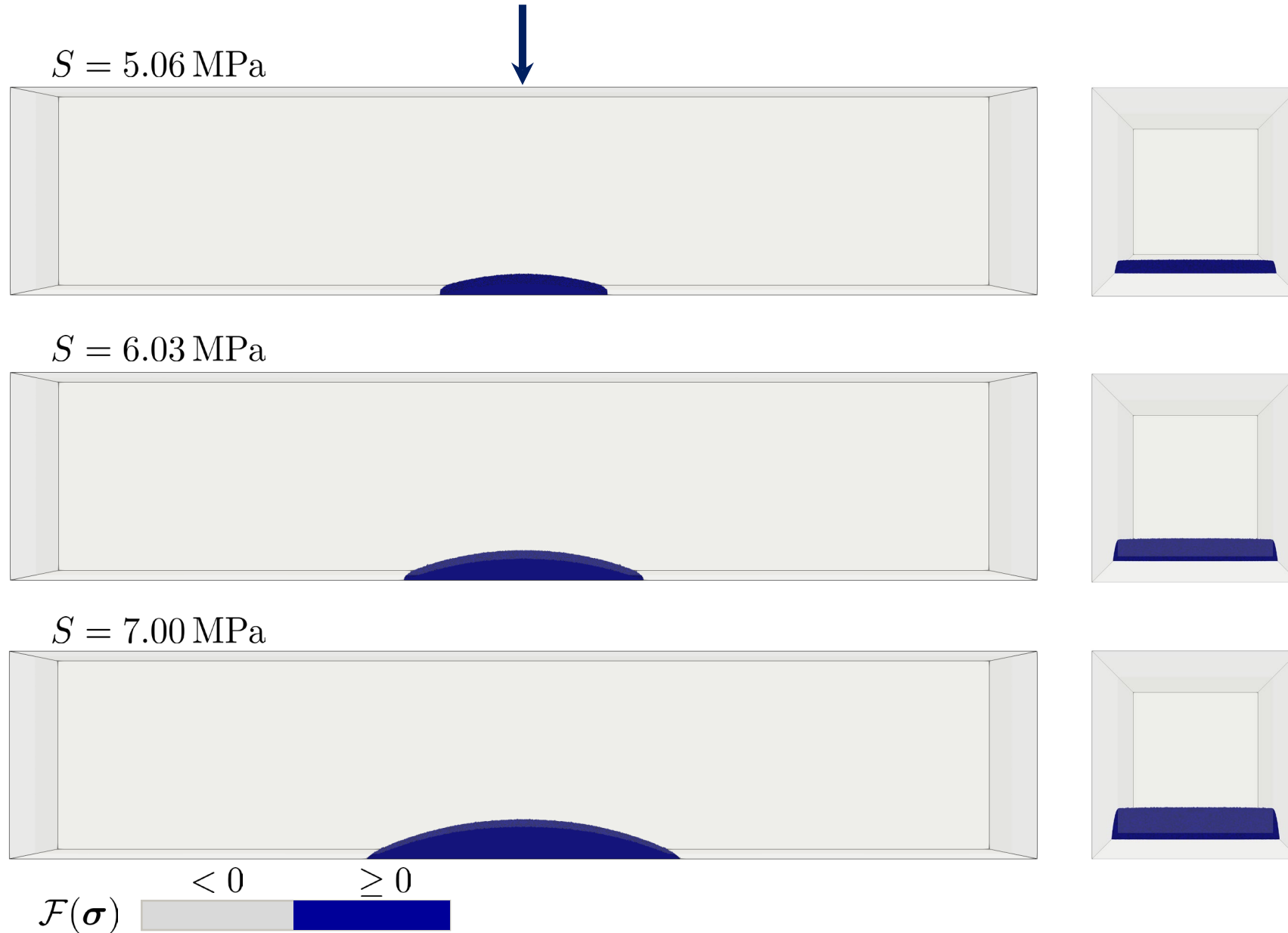
# Complete Theory of Fracture for Elastic Brittle materials, e.g., concrete, glass, cermaics, elastomers, etc.

1. Must respect material separation of length scales
  - min.  $H, W = 10 * D_{\max}$
2. Must violate strength surface  $[F(\sigma)]$  in a region for cracks to nucleate
  1. Necessary but not sufficient condition
  2. Cracks can only nucleate in violation set
3. Energetic competition in the set of strength surface violation between elastic energy +  $G_c * A_c$  or more elastic energy and no crack extension.
4. **Min. material properties:  $\sigma_{ts}, f'_c, G_c, E_c, \nu$**

# Violation of Strength Surface – 4-pt. loading



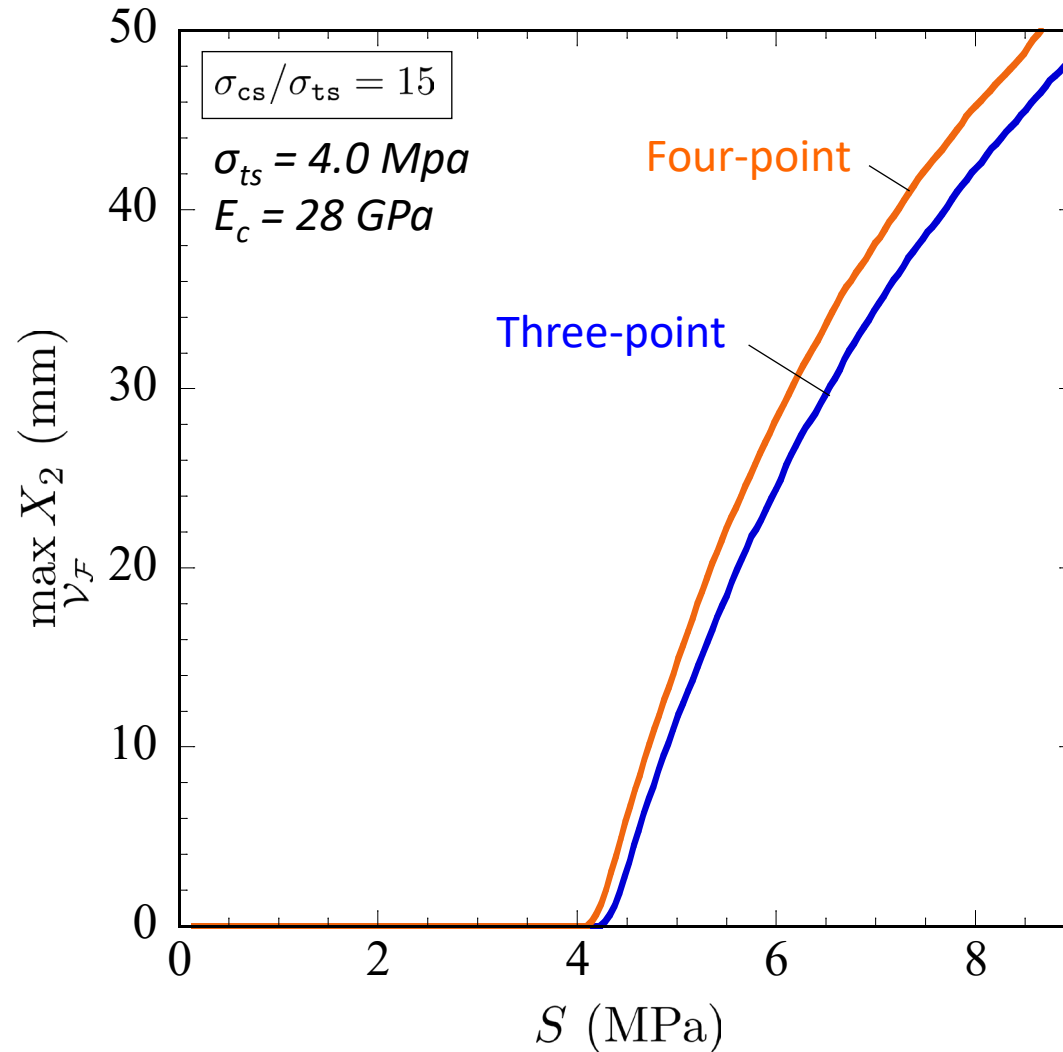
# Violation of Strength Surface – 3-pt. loading



7 MPa ~ 1 ksi

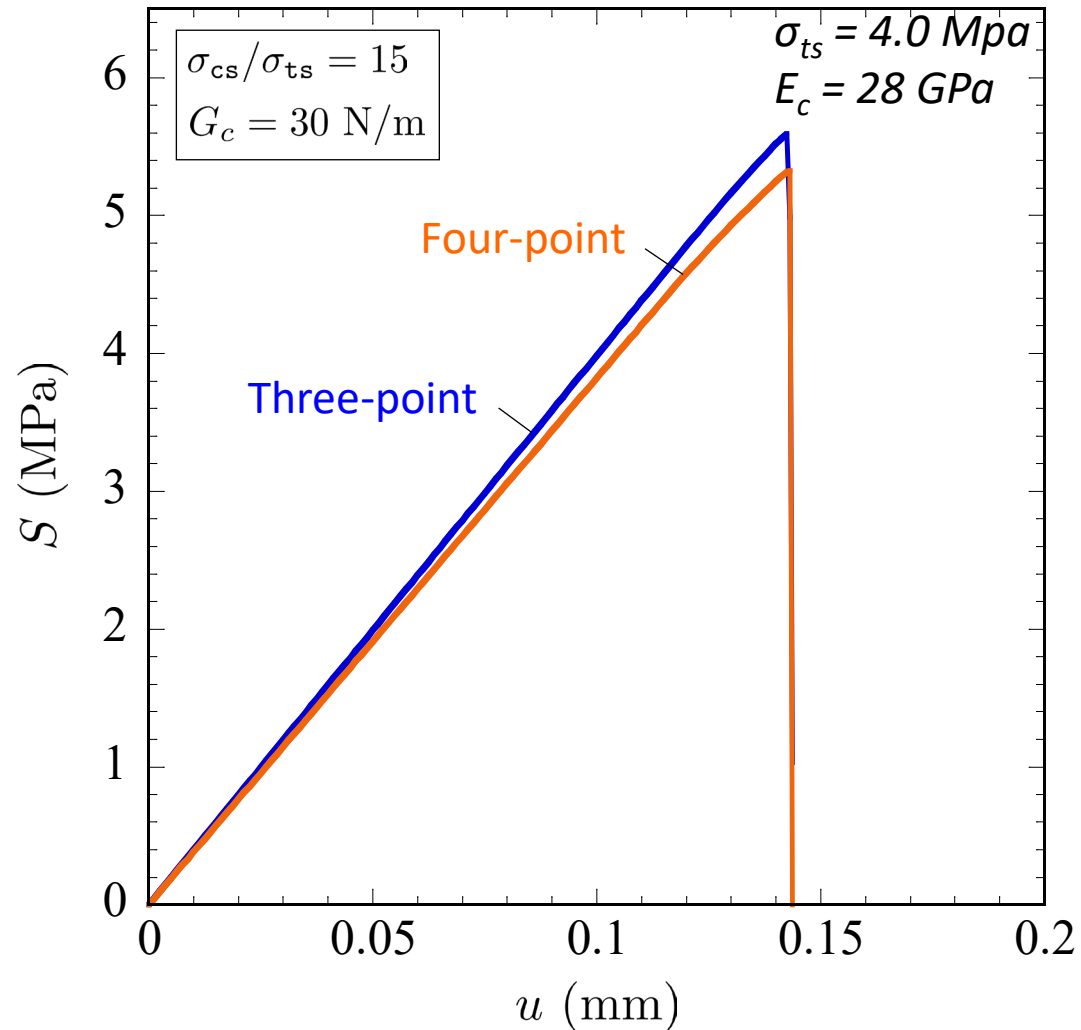
# Strength Surface Violation

## *Beam Height Comparison*



7 MPa ~ 1 ksi

# Difference in 3-pt & 4-pt flexural strength ( $S_{\max}$ )

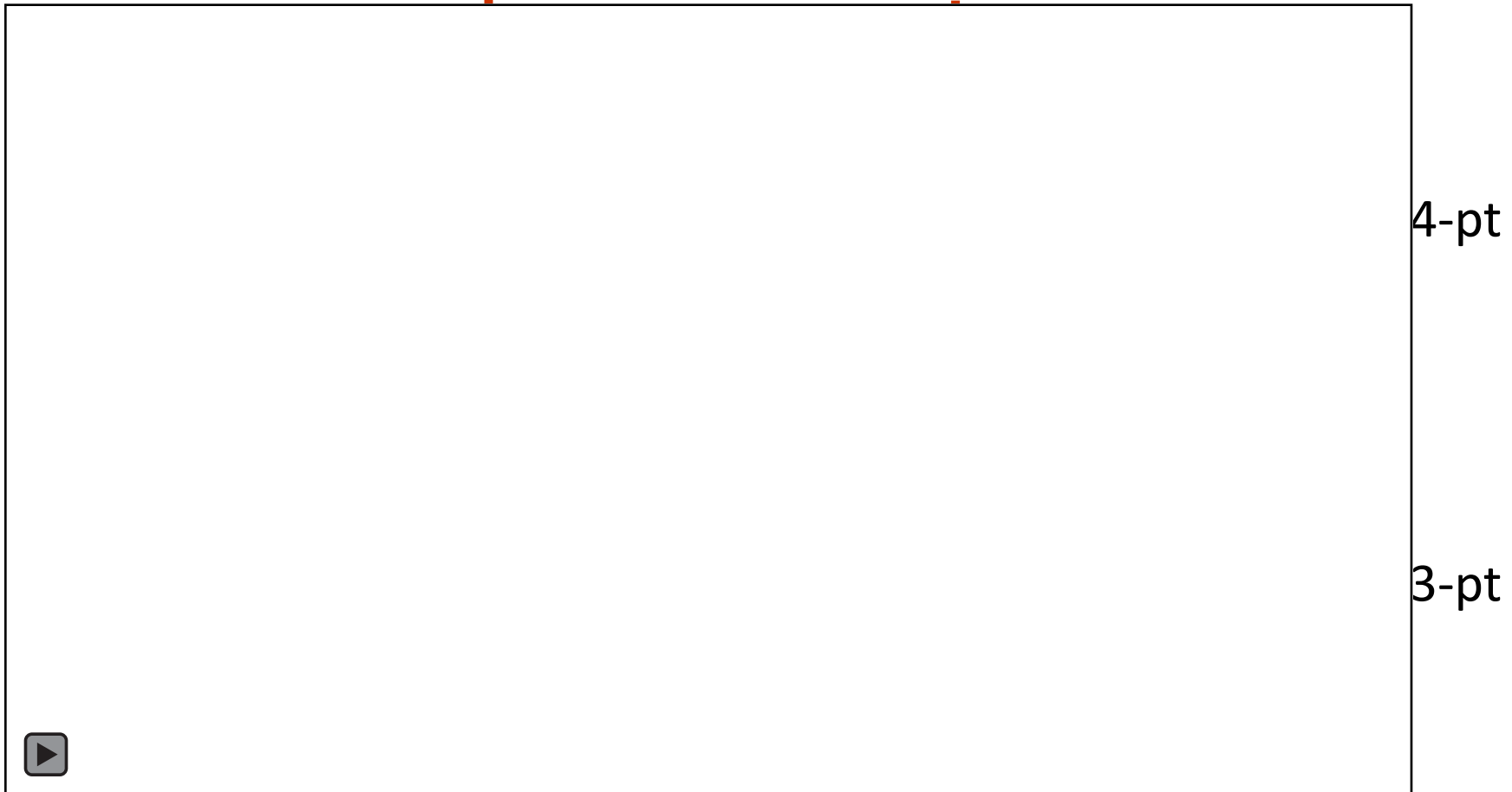


$\sigma_{ts}$  = tensile strength  
 $\sigma_{cs}$  = tensile strength  
 $E_c$  = elastic modulus  
 $G_c$  = fracture toughness

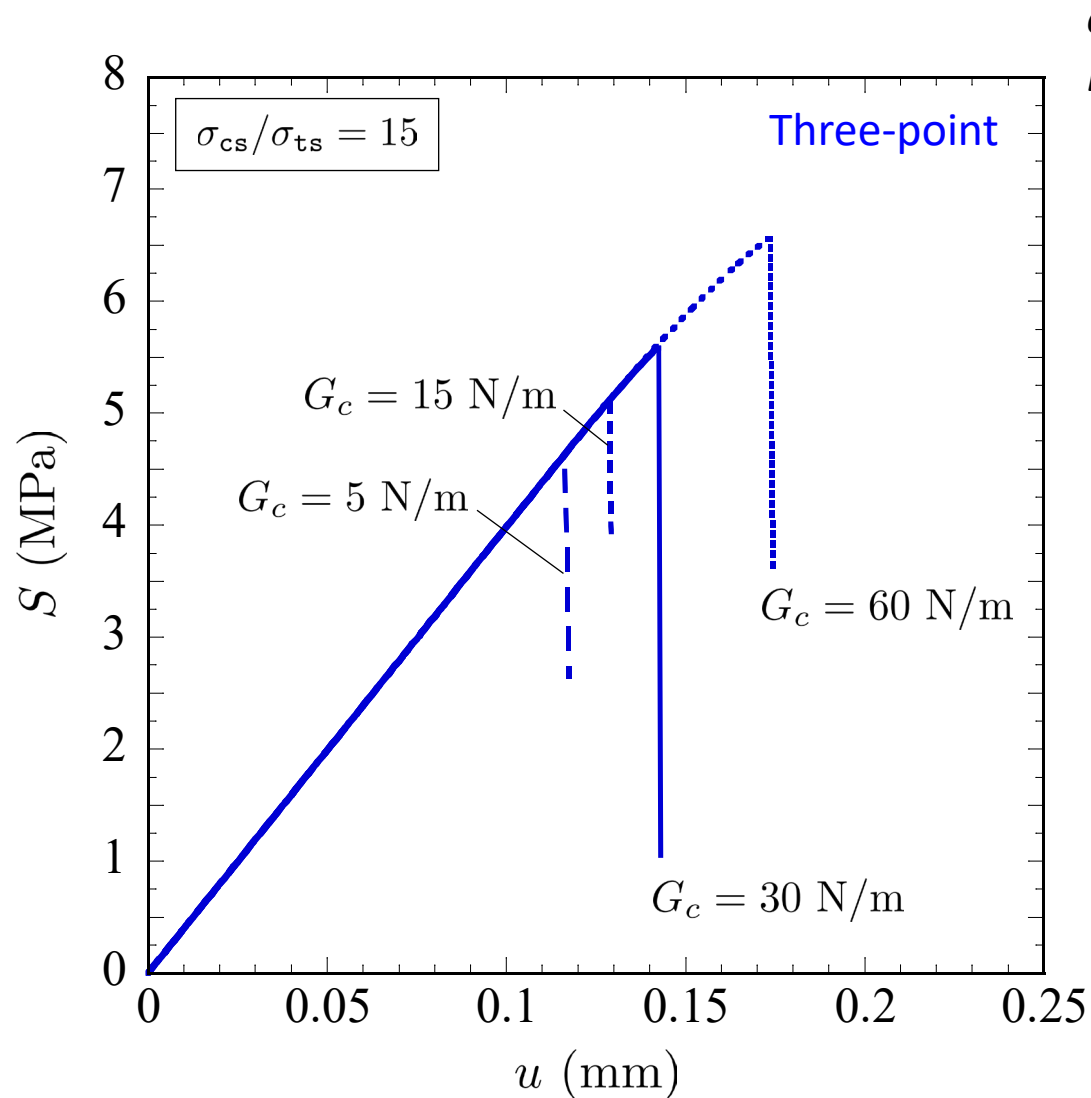
7 MPa ~ 1 ksi

# Crack Nucleation / Propagation

## *3-pt & 4-pt beams*

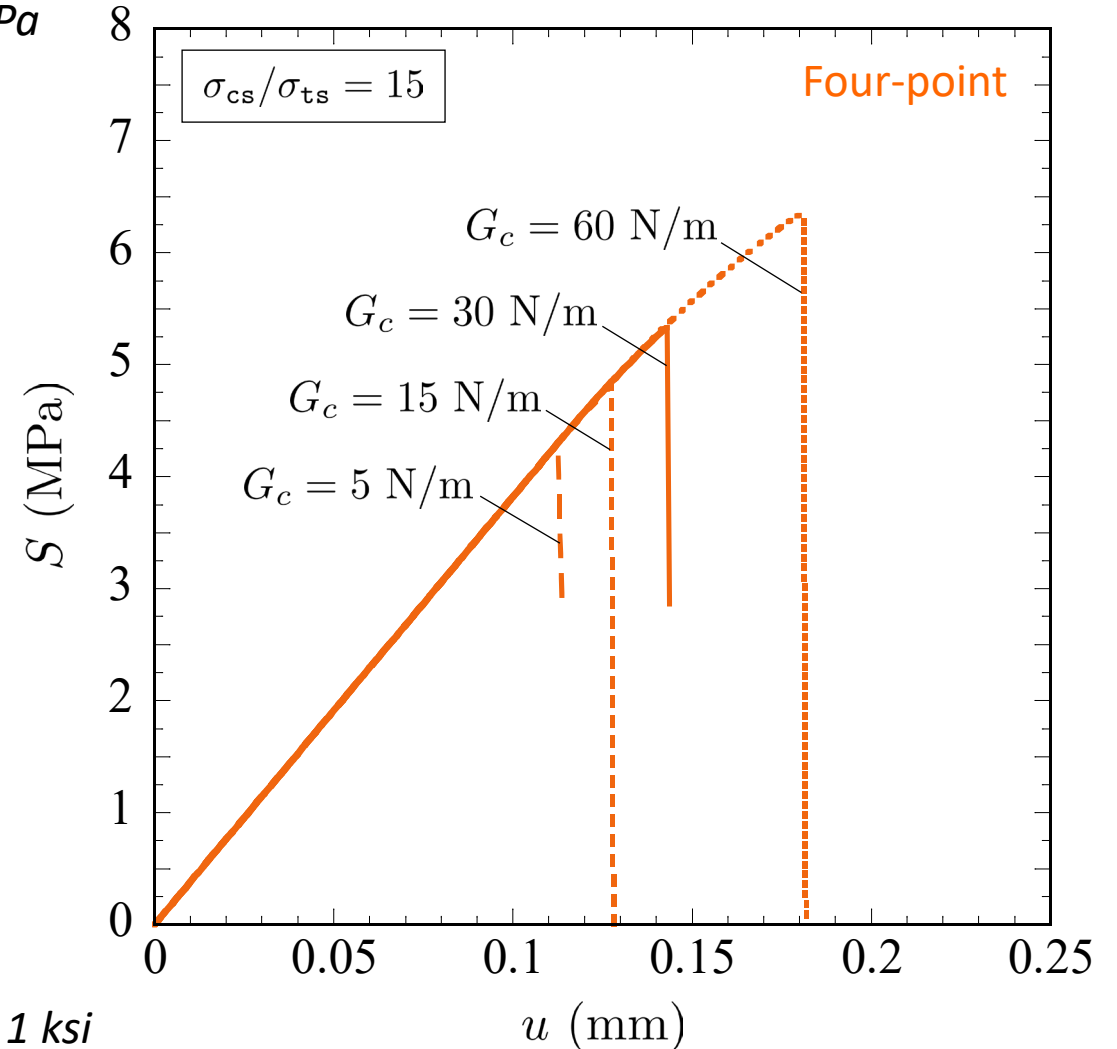


# Comparing $S_{\max}$ for 3-pt vs 4 pt and varying $G_c$



$\sigma_{ts} = 4.0 \text{ MPa}$   
 $E_c = 28 \text{ GPa}$

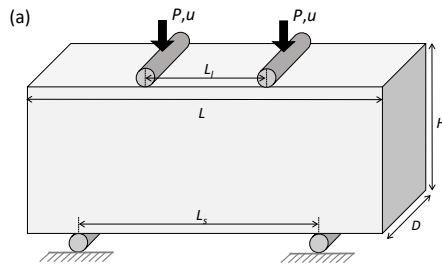
7 MPa ~ 1 ksi



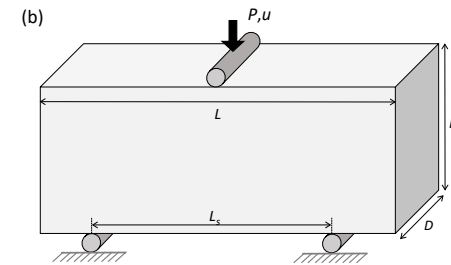
# Predicting Beam “Flexural” Strength

$$S_{max} = S_{max} \left( \frac{L_s}{H}, \frac{D}{H}, H, \frac{L_l}{H}, \sigma_{ts}, \frac{EG_c}{\sigma_{ts}^2} \right). \quad (1)$$

$$S_{max} = g \left( \frac{L_s}{H}, \frac{D}{H}, \frac{L_l}{H}, \frac{EG_c}{H\sigma_{ts}^2} \right) \sigma_{ts}, \quad (2)$$



$$\ell_{ts} = \frac{EG_c}{\sigma_{ts}^2}. \quad (3)$$



$$S_{max}^{4p} = \left( 1 + \alpha \left( \frac{EG_c}{H\sigma_{ts}^2} \right)^\beta \right)^\gamma \sigma_{ts} \quad \text{with} \quad \begin{cases} \alpha = 1.567 \\ \beta = 0.775 \\ \gamma = 0.678 \end{cases} \quad (4)$$

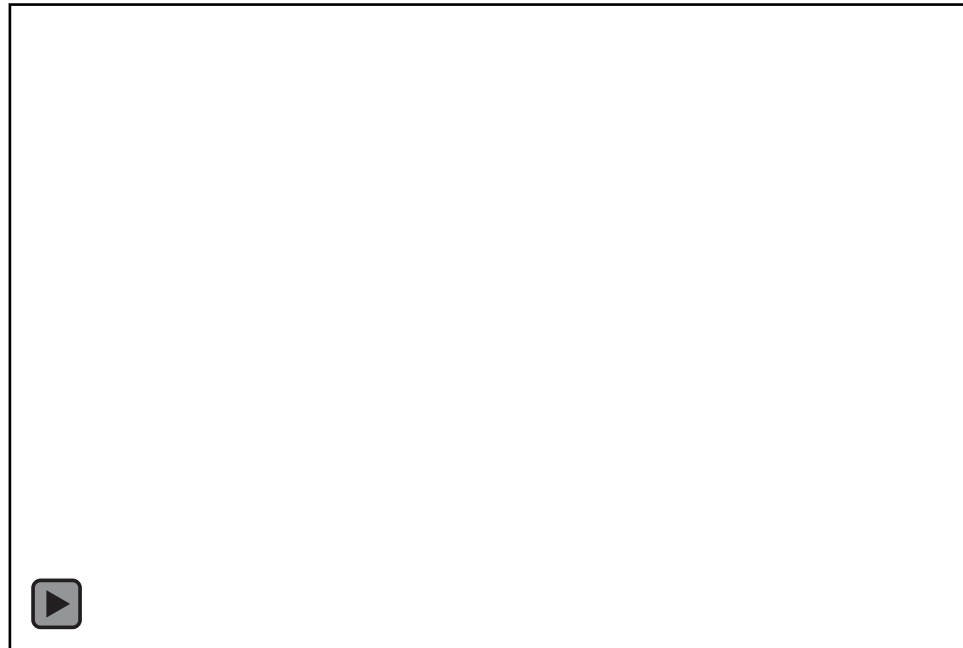
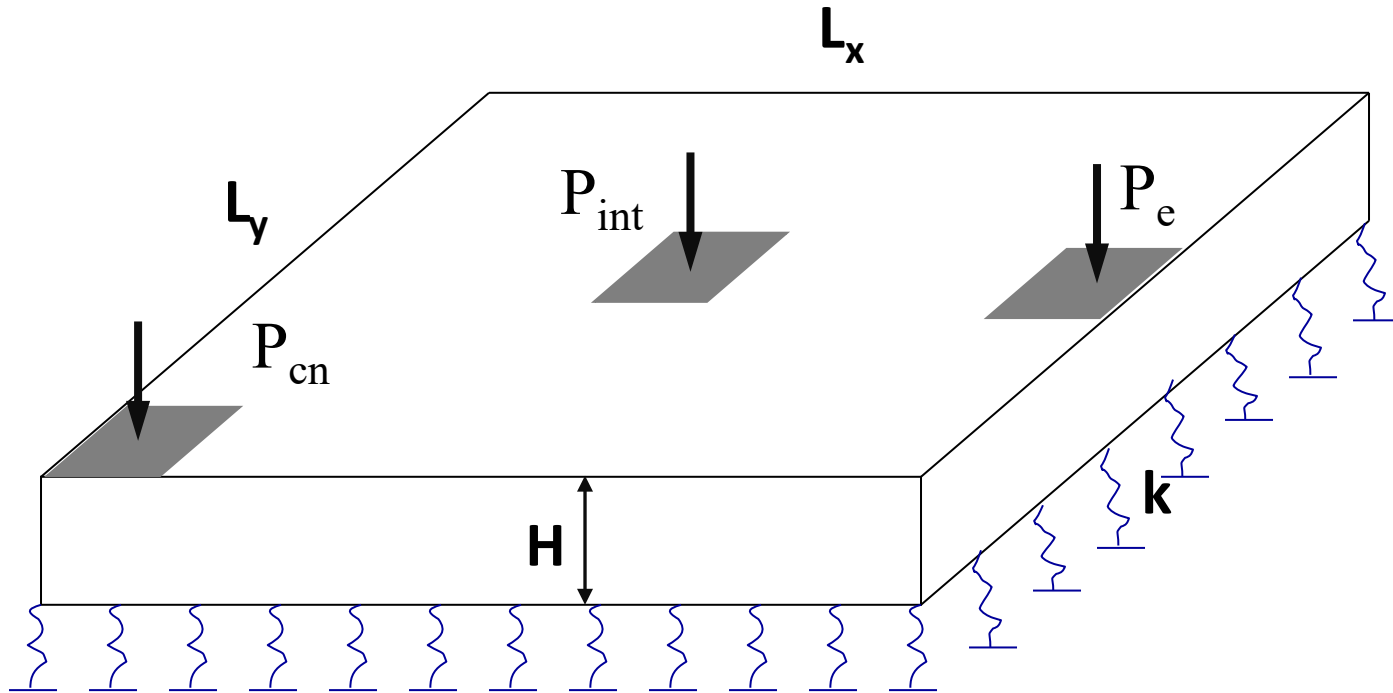
$$S_{max}^{3p} = \left( 1 + \alpha \left( \frac{EG_c}{H\sigma_{ts}^2} \right)^\beta \right)^\gamma \sigma_{ts} \quad \text{with} \quad \begin{cases} \alpha = 1 \\ \beta = 0.621 \\ \gamma = 1 \end{cases} \quad (5)$$

# Summary of 3-pt vs. 4-pt bending test

- For same concrete (w/o voids/defects), 3-pt and 4-pt beam test will give at least 6-9% difference (depends on H, L/H)
  - Higher diff. with lower L/H and material heterogeneity (20%-25%)
- Beam's boundary layer tensile strength and stochasticity greatly affects flexural strength values!!
  - 4-pt reflects minimum  $\sigma_{ts}^{bl}$  while 3-pt reflects the mean  $\sigma_{ts}^{bl}$
- Need to measure  $\sigma_{ts}$ ,  $G_c$ , and  $E_c$  to predict slab capacity

# Slab Capacity – The Goal

$$F_{max}^j = f(L/RRS, a/RRS, L_x/L_y, \frac{\sigma_1}{\sigma_2}, l_{ch}/H) * \sigma_{ts}$$



# Summary of Concrete Strength (Denominator)

- MOR or Flexural Strength is “strength” standard for slabs but has *many limitations*
- Slab “strength” or capacity is much larger than beam strength
  - FRC slab strength > Plain concrete slab strength
- Slab design → Use slab flexural capacity
  - Depends slab geometry, loading, concrete materials, fibers, etc.
- Beam flex. strength (MOR)
  - Depends on  $\sigma_{ts}$ ,  $G_C$ ,  $E$ , and  $\nu$
- Slab capacity calculation is the goal for denominator!

# Acknowledgements

- Slab Tests – many UIUC graduate students (2001-2015)
- Professor Oscar Lopez-Pamies
  - Professor, Civil & Environmental Engineering, University of Illinois
- Subhrangsu Saha
  - Graduate research student, CEE, University of Illinois

# Slab Design Webinar



|               |                                                                  |
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| 12.10 – 12.30 | Juan Pablo Covarrubias   <b>The How &amp; Proof of Concept</b>   |
| 12.30 – 12.55 | Dr. Lev Khazanovich   <b>The Numerator</b>                       |
| 12.55 – 1.20  | Dr. Jeff Roesler   <b>The Denominator</b>                        |
| 1.20 – 1.30   | Nigel Parkes   <b>Curl Data Collection</b>                       |
| 1.30 – 2.00   | Robert Rodden   <b>The SLABS™ Software Demo &amp; Work Queue</b> |
| 2.00 – 2.20   | All   <b>Questions &amp; Answers</b>                             |
| 2.20 – 2.30   | Nigel Parkes   <b>Closing Remarks and Next Steps</b>             |

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## Slab Design Workshop

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